Confidence Interval Worksheet

October 2026

The General Clinton Canoe Regatta is one of the longest canoe races in America. It is 63-miles long and takes several hours to complete. It's actually so long, racers often have "pit crews" at different points along the river. We are going to get a sense of the average time it takes these racers to complete the race.

Our random sample of 150 canoes had an average finish time for the 2025 Regatta of 9.02 hours. The population variance σ^2 is known to be 1.75 hours^2.

For this worksheet, we will assume we have passed all of our assumptions. Below is the z-values for different confidence levels as well as the confidence interval formula.

Conf. Level	99%	95%	90%	80%
Z*	2.58	1.96	1.645	1.28

$$\bar{x} \pm z^* \sqrt{\frac{\sigma^2}{n}}$$

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1. Please build a 95% confidence interval for the mean time to complete the race. The required pieces for the formula are given in the table and the middle paragraph above.

   So first up lets write down the bits of information we know....

   $$\begin{aligned} \bar{x} &= 9.02 \\ \sigma^2 &= 1.75 \\ n &= 150 \end{aligned}$$

   and for 95% confidence

   $$z^* = 1.96$$

   Now we end up with....

   $$\bar{x} \pm z^* \sqrt{\frac{\sigma^2}{n}}$$
   $$9.02 \pm 1.96 * \sqrt{\frac{1.75}{150}}$$
   $$(9.02 - 1.96 * \sqrt{\frac{1.75}{150}},\ 9.02 + 1.96 * \sqrt{\frac{1.75}{150}})$$
   $$(8.80,\ 9.23)$$

2. Please interpret your confidence interval

   We are 95% confident the true mean finish time for the Clinton Regatta is between 8.80 and 9.23 hours.

3. Please calculate a 99% confidence interval for the mean finish time.

   Similar to before but now we are going to use z* of 2.58.

$$\bar{x} \pm z^* \sqrt{\frac{\sigma^2}{n}}$$

$$9.02 \pm 2.58 * \sqrt{\frac{1.75}{150}}$$

$$(8.74,\ 9.30)$$

4. What changed between the two intervals?

   The width of the 99% confidence interval is wider than the 95% confidence interval

5. Please calculate another 95% confidence interval but this time assume our sample size doubled to 300.

   So similar to the first but now $n = 300$...

$$\bar{x} \pm z^* \sqrt{\frac{\sigma^2}{n}}$$

$$9.02 \pm 1.96 * \sqrt{\frac{1.75}{300}}$$

$$(8.87,\ 9.17)$$

6. Please calculate another 95% confidence interval but this time assume our sample size quadrupled to 600.

   $n = 600$

$$\bar{x} \pm z^* \sqrt{\frac{\sigma^2}{n}}$$
$$9.02 \pm 1.96 * \sqrt{\frac{1.75}{600}}$$
$$(8.91,\ 9.13)$$

7. How do the widths of the three confidence intervals change?

   They shrank as the sample size grew

8. Please calculate a 95% confidence interval but assume the mean is 8.5.

   So in this situation we are changing $\bar{x}$ to 8.5 hours.

$$\bar{x} \pm z^* \sqrt{\frac{\sigma^2}{n}}$$
$$8.5 \pm 1.96 * \sqrt{\frac{1.75}{150}}$$
$$(8.23,\ 8.71)$$

9. Please comment on how the confidence interval changes. Focus on both the width of the interval and it's location.

   The width hasn't really changed (if there is any different it's purely a rounding error). But the location (ie means) of the intervals has shifted. The new one is centered (symmetrical around) 8.5 while the old one is 9.02.

10. Finish the sentences below based on your solutions above.

    "If the confidence level increases, the width of the confidence interval will ....." **increase**

"If the sample size increases, the width of the confidence interval will ....." **decrease**

"If the sample size increases by a factor of 4, the width of the confidence interval will ....." **decrease by half**

You don't *need* to know this but it's just a fun math thing. Take the formula we had when we started with

$$\bar{x} \pm z^* \sqrt{\frac{\sigma^2}{n}}$$

but now we are going to assume that our sample size has quadrupled to $4n$ so we replace n with $4n$ to get...

$$\begin{aligned}
\bar{x} &\pm z^* \sqrt{\frac{\sigma^2}{4n}} \\
\bar{x} &\pm z^* \sqrt{(\frac{1}{4})\frac{\sigma^2}{n}} \\
\bar{x} &\pm z^* \sqrt{\frac{1}{4}}\sqrt{\frac{\sigma^2}{n}} \\
\bar{x} &\pm \frac{z^*}{2}\sqrt{\frac{\sigma^2}{n}}
\end{aligned}$$

so the margin of error ($z^* \sqrt{\frac{\sigma^2}{n}}$) will be cut in half. There is a decreasing return on increasing your sample size. Intuitively if you have five participants, getting a sixth is a big deal. If you have 5,000 then getting 1 more isn't that important for your results.

"If the sample mean increases, the width of the confidence interval will ....." **stay the same**

"If the sample mean increases, the *location* (middle) of the confidence interval will ....." **increase. The confidnece interval is centered at the sample mean.**