Exam 2 Overview Math 104

October 2026

This exam will be similar to the first exam. There will be some general knowledge questions, a section on hypothesis testing (probably a z-test but possibly a permutation test), a section that has a confidence interval. There will be a less math-y section that talks about sampling methods.

1 Executive Summary

- Be able to do a z-test
- Be able to do a confidence interval
- Be able to interpret a confidence interval or z-test
- Know your 3 assumptions (random, IID, normal or large n)
- Give an example of different sampling methods (simple random sampling, stratified, etc..)
- Be able to identify biases in surveys (eg undercoverage)

2 Permutation Test

- Gives exact probabilities and not approximations
  - Eg there is only 3 ways in 100,000 that I could deal a hand to myself that nice
- Won't be tested nearly as in-depth as the z-tests but still worth knowing
- What *is* a null and alternative hypothesis?
  - Alternative hypothesis is what we are trying to show
    - Eg this jury wasn't picked randomly
    - Eg the proportion of adventures that Indiana Jones saves the world is great than .75
  - Null hypothesis is everything the alternative isn't and is usually the not fun "default"
    - Eg the jury was randomly selected
    - Eg the proportion of adventures Inidiana saves the world is less than or equal to .75
    - Eg our new medication isn't different than the old medication

- A permutation test....
  1. writes out all possible outcomes
  2. finds the proportion that fulfill some crtierion
  3. Can assign an exact probability we see an (EVENT/Success)
  4. Based on if that probability is way low we can call bs
- Eg In a business of 40 people with 5 employees related to the owner, what is the probability all three promotions would go to the owner's relatives if promotions were handed out randomly?
- No, you will not need to know how to calculate the above
- Useful when possible but computationally slow and intensive

3 Normal Distribution

You will not need a computer or internet in the exam and relevant information will be given for any values that you'll need for the normal distribution. Focus mostly on...

- Being able to sketch a normal distribution and...
  - indicate the mean and variance for the distribution
  - Shade in parts that correspond to probabilities..
    - Eg P(X < -1) is everything to the left of -1 on the normal distribution
    - Eg P(X > -1) is everything to the right of -1
    - Eg P(-1 < X < 3) is everything between -1 and positive 3
- Understand that many distributions are not actually normal but kinda look normal
  - This is why normal approximations work
  - Eg binomial is X heads in Y coin flips, and is usually bell shaped
- Be able to standardize a variable:

$$\frac{\text{(Observation)} - \text{(Population Mean)}}{\text{(standard deviation)}}$$

- And used standardized variables to compare apples to oranges
  - Eg this apple's standardized weight vs this orange's standardized sugar concentration is valid
  - Standardized values give how many standard deviations above/below the mean the observation is
  - Eg the apple is 2 standard devaitions above the mean weight of apples but this orange is only 1.5 st. dev. above the mean sugar concentration

4 Sampling Distributions

- Be able to define a sampling distribution
  - The long term behavior of a sample statistic, ie if we take a sample and recorded the mean and did it again a million times
  - Standard error: the standard deviation of the sampling distribution (idk why it gets a special name)
- Understand the sampling distribution talks about some summary statistic from the data and not the data itself
  - It won't help you much understand where your next observation will be
- Population vs Sample vs Sampling distributions
  - Population dist is everyone; usually unknown but hopefully looks like the sample
  - Sample dist is the data we collected; only fully known distribution we have
  - Sampling dist is the behvior of the summary statistic; we know it if our assumptions are met
  - Parameter: numeric summary of the population; unknown but hopefully similar to the sample statistic
  - Statistic: numeric summary of our data (our sample); we know this value

5 Z-tests

- Z-test is used when the population variance is known (rare)
- Assumptions for the sampling distribution
  - Random: Our data was collected randomly if it's a survey or the treatments were randomly assigned for an experiment
  - Independent and Identically Distributed:
    - Independent: Knowing something about one observation doesn't tell you anything about the next observation; ie one observation doesn't influence another observation
      - Eg Genetically related subjects aren't independent
      - Eg Measuring the same things over time (eg your weight daily) won't be independent
      - Rolling two dice should be independent
    - Identically Distributed
      - No a priori belief that one observation will be different than another observation (outside of random noise)

        - Basically there is no subpopulation or lurking variables that might be messing up our analysis
        - Eg Testing high jump distances given different types of shoes....but the group for one type of shoe is made up of middle aged math profs while the other group with a different type of shoe is the local college's men's basketball team
        - Super common red flag is if the sample distribution is bimodal or appears to form groupings
    - Population distribution is normal or n is large
        * Don't know population's shape but we do know the sample's shape so use that instead
        * How large n needs to be depends on the shape of the sample distribution, see slides on the z-test for details
        * Large n invokes the Central Limit Theorem which says a sampling distribution will approach normality if the sample size is large

- Understand how strong a p-value is classified by
    - See z-tests notes for details of the cut offs
    - I'll give you the table on the exam
- Using a p-value be comfortable writing a decision
    - You *will* lose points if you say reject/fail to reject
    - Strength of evidence is what we want
- Define a p-value
    - Probability we'd see a sample statistic as or more extreme than what we saw, given the null hypothesis is true
- Understand the limits of a p-value
    - Doesn't give the probability the null/alt hypothesis is true
    - A large p-value doesn't mean the null is true
    - Just one piece of evidence among several in an analysis (eg graphing is another)

6 Confidence Intervals

- Same assumptions as for hypothesis testing
- Allows us to find a range of reasonable values for where we think the true parameter is
- Define "confidence"

  - It's the coverage rate for our intervals...90% confidence means about 90% of our confidence intervals will contain the parameter
  - That requires us to take many, many samples and for each sample we build a confidence interval
- Interpret confidence intervals
- Confidence intervals talk about the location of the mean
  - They do NOT talk about where we think the next observation will be (it exists but is called a prediction interval; not discussed)
  - They do NOT talk about the probability the mean is within the interval
    * The mean is or isn't inside the interval so there is no repetitions so no frequency so no probability
    * Remember that probability is the long term frequency of an event...it can't be applied to a yes or no question without repetitions

7 Sampling Methods

First sampling...

- Difference between prospective and retrospective sampling
  - prospective follows things over time
  - retrospective collects info from the past
  - prospective is preferential to retrospective but can be too costly/unreasonable
- Simple Random Sample
  - Give a brief definition
    * everyone is equally likely to be picked
  - Or give a small example
- Stratified Sampling
  - Give a brief definition
    * We do a simple random sampling within each subpopulation
  - Or give a small example
  - Why this might be picked
    * Often because a given strata (subpopulation) is hard to find if you don't go looking for them (eg trans students on campus aren't likely to crop up in a survey of 15 students)

        - Or you really are interested in comparing subpopulations so want them equally represented

- Clustered Sampling
    - Give a brief definition
        - We randomly sample tiny tiny subpopulations (cluster) and record all subjects in that cluster
    - Or give a small example
    - Why this might be picked
        - It's a mix of random (which clusters?) but also convenient in that once a cluster is identified you just study to everyone in that cluster
        - Eg

- Snowball Sampling
    - Brief definition or example
        - One participant introduces you/leads you to the next participant
    - Loses independence in your sample which is bad
        - Say a rare type of bee leads you back to it's colony....of related bees
    - Often done in desperation

- Convenience Sample
    - define or give an example
        - You sample the easiest subjects
        - Eg I text my friends from undergrad who were in the same political club their views on (political thing)....easy to get the sample and I can already tell you what the survey results will be
    - Often leads to undercoverage and a lack of independence
    - Is bad

- Know your biases
    - Undercoverage: some subjects will never be chosen for your sample
    - Wording of Questions: questions made too confusing or "leading" by being emotionally charged
    - Response Bias: People are taking your survey for a reason and that might be problematic
        - Zealous on a politcal topic
        - Also, the bias that covers people lying or misremembering
    - Non-Response Bias: People refuse to take your survey....is there a reason for this?
        - Eg The survey topic is too "heavy" and makes students uncomfortable or brings up painful feelings

    - Eg Think about trying to get the views of Richard Nixon's "silent majority" who won't take polls

8 Pretentious Questions To Expect

1. I'm testing you on these because these are used as litmus tests to grade people are statisticians. A couple questions cannot encapsulate's someones understanding of statistics but since you will be judged for them I might as well "teach the test"
2. What's a p-value?
    - The probability we would see a sample statistic this extreme if the null hypothesis was true
3. What's the difference between st. deviation and st. error?
    - st deviation is the square root of variance
    - st error is the st. deviation of the sampling distribution

9 Table

Test Name	σ^2 known?	Assumptions	Relevant Distribution	Test Statistic	Conf. Interval
z-test	yes	(Random), (IID), (Large n or the pop is normal)	Normal	$\frac{\bar{x}-\mu}{\sqrt{\frac{\sigma^2}{n}}}$	$\bar{x} \pm z_{1-\alpha/2}\sqrt{\sigma^2/n}$