Z-tests and Confidence Intervals: Homework 6

NAME:

CLASS START TIME:

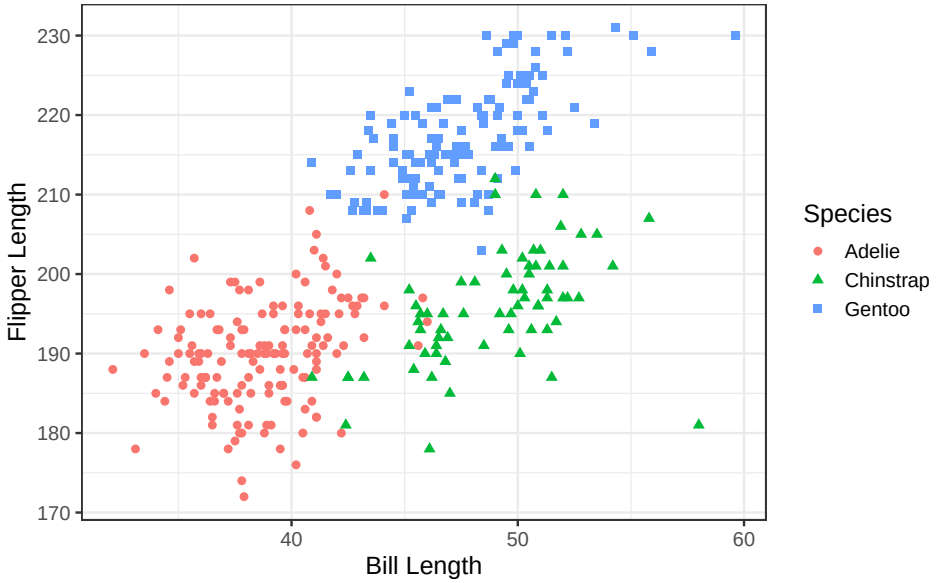
1 Background

We will be working with the penguin data again. In particular we are going to be running a couple z-tests and confidence intervals around the mean bill lengths for the different species of penguins.

As background, in the early 2000's 344 penguins were randomly sampled from some islands off the coast of Antarctica. The penguins came in three distinct species: Adelie (152 penguins), Chinstrap (68 penguins), and Gentoo (124 penguins). The number after the species is the number of that type of penguin in our sample.

1. Please, off on the side, comment on the form, direction, strength, and if there are outliers in the below scatterplot. It may be useful to discuss the data by species.

   The graph, ignoring species, is moderately strong, postiive, with at least one outlier (far bottom right). It's roughly linear. When looked at by species, Adelie penguins appear to be more of a cloud of points while Chinstrap and Gentoo still look moderately strong, uphill and linear.



2. Based on the above graph do you believe...

   A. **TRUE** or **FALSE**: Gentoo have the highest flipper and bill length on average
   FALSE, there seems like Chinstrap might have a slightly higher bill length. At the ver least Gentoo isn't the longest

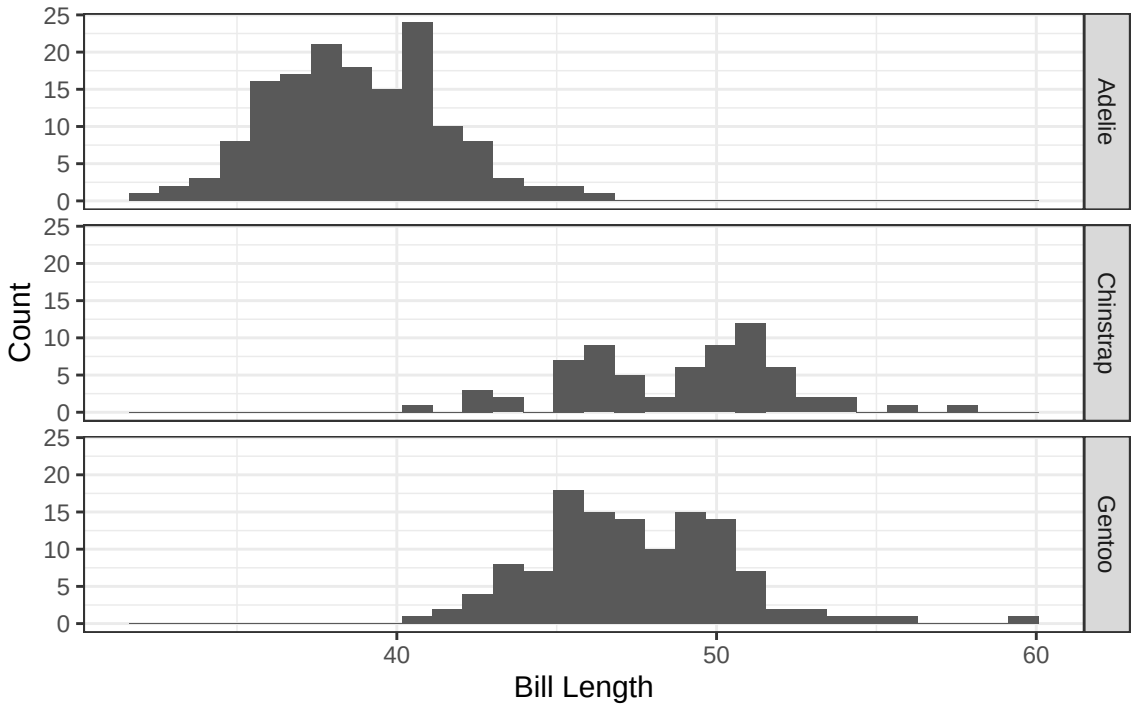
   B. **TRUE** or **FALSE**: The average bill length for the Chinstrap and Gentoo penguins are approximately the same
   That's roughly true.

C. **TRUE** or **FALSE**: Adelie penguins seem to have the shortest flipper and bill length on average.
TRUE, they are solidly in the bottom left of the graph which indicates smallest flipper and bill length.

D. **TRUE** or **FALSE**: Species doesn't seem to have an effect on bill or flipper length.
FALSE. Depending on the species my expectation of things like bill length changes (e.g. if a penguin has a bil length of 50 I suspect it's either chinstrap or gentoo but not Adelie.

2 Confidence Intervals

I talked with a friend who really knows penguins and was assured that, for the length of penguin's bills, the population variance, σ^2, was 10 (units are mm^2 but don't worry about that). Assume it's the same variance regardless of the species. Below are the histograms of the bill length for each species of penguin ploted as a histogram.



3. Please describe the three histograms below. Be sure to mention if they are symmetric, how many modes, and if there are outliers. Also identify if you think it is approximately normally distributed (bell shape).

   Each one is bimodal although it's less obvious for Adelie penguins. They are very roughly symmetric and Chinstrap and Gentoo have outliers.

4. Time for assumptions: For each of the three species of penguins, please indicate if the population distribution is normally distributed or if the sample size is sufficiently large.

   The populations aren't normally distributed because the sample distributions aren't normally distributed. They should look the same so we think the populations will be roughly bimodal. Our sample sizes though are relatively large for symmetric graphs so we pass this assumption via the Central Limit Theorem (sufficently large sample size)

5. Time for assumptions: For each of the three species, was the data randomly collected? We will tackle this in parts below.

   Yes, the beginning instructions say they were randomly selected.

   Time for the assumptions: Was the data independent and identically distributed?

6. Is the data independent? That is, do we believe one penguins bill length should be independent of another penguins bill length?

   They are probably independent. The two biggest issues would be genetic relatives and the possiblity that the penguins were competing for resources. Personally I'm not too worried about either so long as the penguin colony was sufficently large (eg 1000+). My thinking is that relatives would be a bit more rare and at that size, presumably, the enviroment can sustain those populations.

7. Is the data identically distributed? HINT: focus on the number of modes for each species. No, the data is not identically distributed. Each species is, roughly, bimodal which raises concerns about the data being identically distributed.

8. If there is reason to believe there are subpopulations within each species, what variable could be driving the discrepancies we are observing?

   Probably the sex of the bird. One mode might be the male penguins and one might be the female penguins. We would need to graph the data by sex to get a better sense of what's happening.

9. Assume we passed the assumptions. Please build a 95% confidence interval for the mean for each penguin species. Below is a table that gives you relevant information you may want.

Species	Sample Mean	Variance	Sample Size
Chinstrap	48	10	68
Gentoo	47.5	10	124
Adelie	38	10	152

$$\bar{x} \pm z^* \sqrt{\frac{\sigma^2}{n}}$$

$$48 \pm 1.96\sqrt{\frac{10}{68}} = (47.25, 48.75)$$

$$47.5 \pm 1.96\sqrt{\frac{10}{124}} = (46.94, 48.06)$$

$$38 \pm 1.96\sqrt{\frac{10}{152}} = (37.5, 38.5)$$

10. Please interpret the confidence interval you made for the mean of the Gentoo penguins.

    We are 95% confident the true mean bill length for Gentoo penguins is between 46.94 and 48.06 mm long.

11. Note how some confidence intervals overlap and some do not. What do you think this implies about our ability to distingish the two groups based on their average bill length?

    If the confidence intervals overlap it's probably difficult to find a statistical difference. Intuitively a confidence interval is where we expect the truth to be so if two confidence intervals overlap they may actually have the same true value.

12. Do you think the earlier assumption about the penguins having the same population variance (spread of the data) is tenable? HINT: Use the three histograms from earlier.

    Yes, actually. The three histograms have roughly equal spreads whcih implies they have roughly equal variance.

3 Z-tests

Going back to my friend, he also says he knows for a fact the true bill length of chinstrap penguins is actually 48.32mm (which we take as true). Let's run a z-test to see if the mean bill length of Gentoo penguins is also 48.32mm or if it is different from that number. NOTE: We are NOT testing anything with Chinstrap penguins but instead are testing Gentoo penguins' average bill length.

13. Please write down the null and alternative hypothesis. Use μ for the population mean of Gentoo penguins since we are talking about means please.

$$H_0 : \ \mu \ = \ 48.32$$
$$H_A : \ \mu \ \neq \ 48.32$$

14. Let's assume we pass the assumptions (which we already checked in the confidence interval section). Please write down the sampling distribution for the sample mean bill length for Gentoo penguins. Please use the formal notation like in the slides please.

$$\bar{x} \ \sim \ N(48.32, \sqrt{\frac{10}{64}})$$

15. Please find your test statistic (i.e. standardize your sample mean on page 3 using the above sampling distribution).

$$\frac{(47.5 - 48.32)}{\sqrt{\frac{10}{64}}} \ = \ -2.074$$

16. Using the normal calculate online, please find the p-value for this test. That is, please find the probability we would see something more extreme than what we saw if the null hypothesis is true.

    The p-value works out to be .038

17. Using the above p-value, please state your decision for this test. NOTE: If you write "reject" or "fail to reject" you are doing it wrong and will receive no credit. See my notes for a discussion.

    We have moderate evidence the true Gentoo bill length is different from 48.32 mm.

18. Given your earlier confidence intervals, does this surprise you?

    No, the Gentoo confidence interval went from 46.94 to 48.06mm which is where we think "reasonable" values are located. 48.32 is outside of that range so it looks "unreasonable". So moderate evidence sounds about right (it's not that far outside the interval after all).

19. Does Adelie penguins have the same average bill length as Chinstrap as 48.32mm? Answer this question without using a z-test. HINT: Use the graphs from earlier.

    Graphically the answer is no. It appears the longest Adelie penguin bill was only about 46 or 47 mm. We have no data past that so it's unreasonable to beleive the population mean is that high.