Probability Homework

November 2025

You may submit your solutions as a pdf produced by an R markdown, type your response in a textbox on top of this pdf, or you can submit a scanned copy (in pdf format) of hand written solutions. All are acceptable so long as it's in a pdf format.

The homework is broken into three parts. One will be on basic probability; one will be on the normal distribution; and one will be a "stretch" goal.

1 General Probability

We will assume we have a bag full of marbles and cubes. Assume we have 100 object in total in the bag with the following counts

Color	Marble	Cube
Red	10	20
Blue	10	30
Green	20	10

1. What is the probability of drawing a cube?
   Ie what is P(X = cube)? where X is the result of a single random draw.

2. What is the probability of drawing something green AND it being a cube?
   Ie What is P(X = green $\cap$ cube)?

3. What is the value for P(X = green)?

4. What is the probability of drawing a green marble, given that we have drawn a marble?
   Ie What is P(X=green $\cap$ marble |marble) = P(X=green | marble)?

5. What is P(X = Blue $\cup$ Cube)? That is, what is the probability of drawing something that is either blue or a cube?

6. What is P(X=marble)?

7. What is the P(X=red)?

8. What is the P(X = red marble |red)?

9. What is P(X = red marble |marble)?

10. Multiple your answers to questions 8 and 7, then divide by your answer to question 6. How does this compare to question 9? (This is called Bayes' Theorem)

2 Normality

The following deal with the normal distributions and probabilities associated with them. I would advise using the pnorm() function in R when I ask you to calculate probabilities. Note that you will want to tell the function the mean, sd, and whether you'd like probability on the left (lower.tail = TRUE) or right (lower.tail = FALSE). See the help page for the function for more details; or talk with me.

For this question we will be using the NFL data from class. Specifically, it can be approximated with a normal distribution with mean 25 and a standard deviation 10.

1. Please write the above normal distribution in the notation we used in class (vide[1] slide 8 of the Normal Distribution). NOTE: Variance = (St. Dev.)^2

2. The only time I've gotten angry at a football game was when the Packers (my favorite team albeit I'm not a huge NFL fan), blew a 19-7 lead with 2 and a half minutes left in the NFC championship in 2015. Ending score was Seahawks 28 - Packers 22. Standardize the Seahawks' final score.

3. For the above normal distribution with mean 25 and standard deviation 10, use the 68-95-99.7 % rule to estimate the two values where the middle 95% of our data lies between.

4. For the above normal distribution, find the probability the winning team scores more than 45 points. HINT: You'll want to use R for this and most of the remaining questions on Normal Distributions.

5. For the above normal distribution, find the probability the winning team scores less than 5 points.

[1] Latin for "see", as a point of reference/information

6. Sketch a normal distribution with mean 25 and standard deviation 10 with the two areas you just found in questions 4 and 5 shaded in.

7. Find the probability a winning team scores more than 5 and less than 45 points. Ie find the probability a winning team scores between 5 and 45.

8. Find the probability a winning team scores less than 5 or more than 45 points.

9. Calculate how far off the 95% rule is. That is, what is difference between the rule of thumb of 95% and what you calculated in question 8.

10. Write the probabilities found in questions 4, 5, 7, and 8 in notation used in class with P(X=A), $A \cap B$, A^C, $A \cup B$, etc... You do NOT need to rewrite the whole entire question.

3 Exponential Distribution

Do you have a favorite probability distribution? I do. It's the exponential distribution and it's relative the geometric distribution. We are going to push your ability to deal with calculating probability.

The probability (density) function, for $x > 0$, is...

$$e^{-x}$$

for an exponential variable who's mean value is 1. An example of what exponential distribution's probability density function look like is available here.

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1. Calculate the indefinite integral with respect to x in the above function. HINT: A nice safety check is to evaluate the integral from 0 to ∞, which should equal 1.

2. Using your answer to question 1 and the fact probability is the the area under the curve, what is the probability we see an observation between 0 and 1?

3. What is the probability we see an observation greater than 3? HINT: the upper bound for the integral will approach infinity

4. What is the probability an observation that is randomly grabbed will be between 1 and 3?