Correlation

Association between 2 Quantitative Variables

Grinnell College

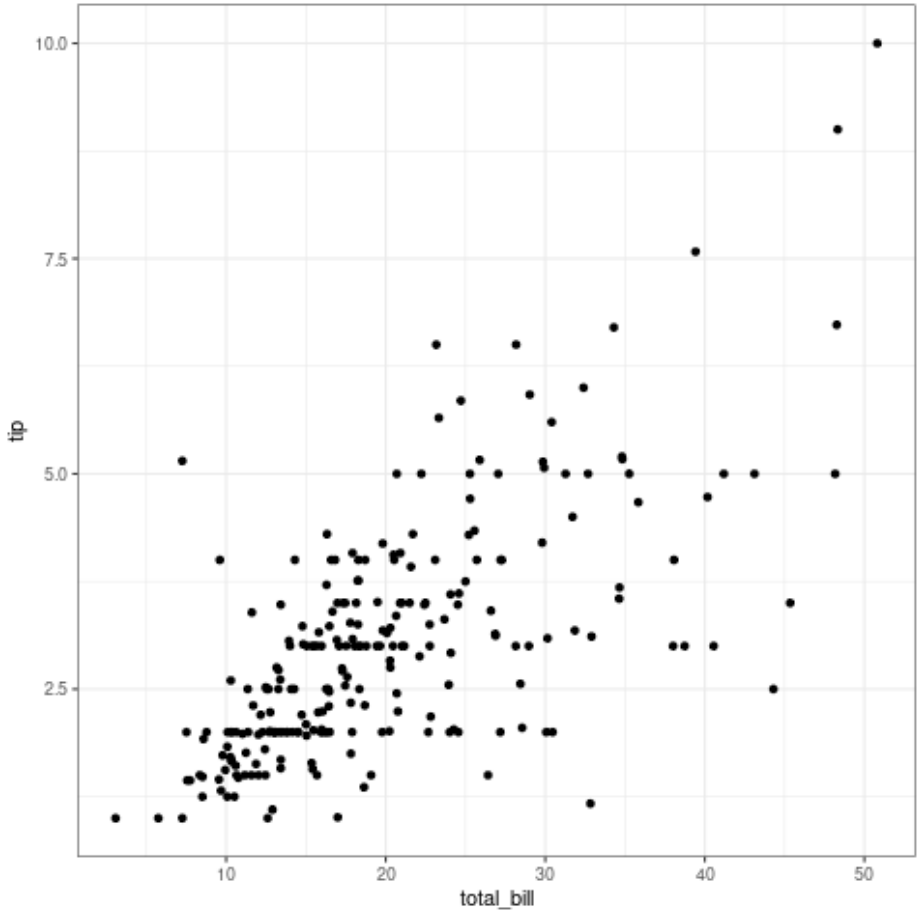
September 22, 2025

Today

- Brief review of order statistics vs moment statistics
- Correlation
- Graphing?

Review – Scatterplots

When we want to plot two quantitative variables $\rightarrow$ scatterplots



Scatterplots let us see if there are *associations* between quantitative variables

Review – Scatterplots

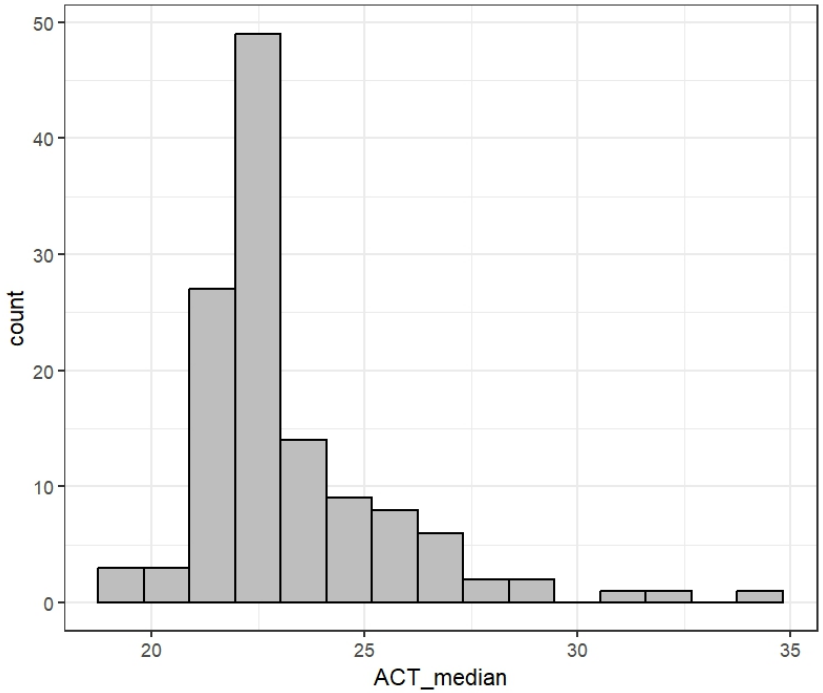
Describing associations in scatterplots:

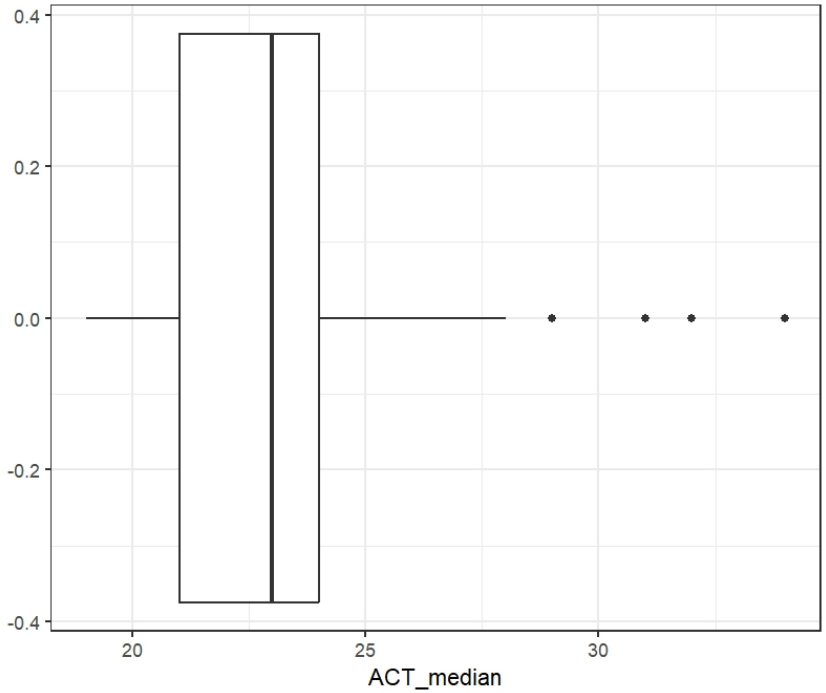
- **Form**: pattern? (linear / non-linear / cloud of points)
- **Strength**: weak / moderate / strong
- **Direction**: positive / negative
- **Outliers**

Extra on Outliers: 1 Numeric Variable

Two ways to look for outliers:

- histogram → gaps in between the bins (preferred)
- boxplot → points outside the 'whiskers'



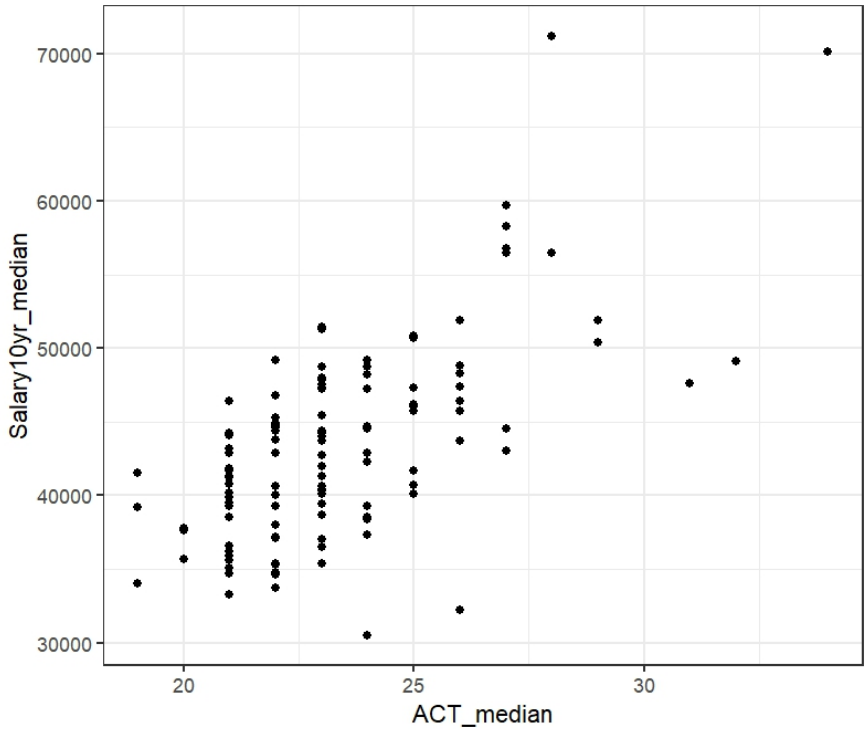


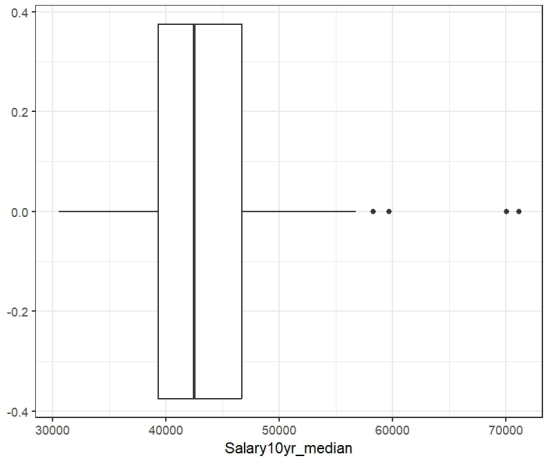
Mention which graph you used!

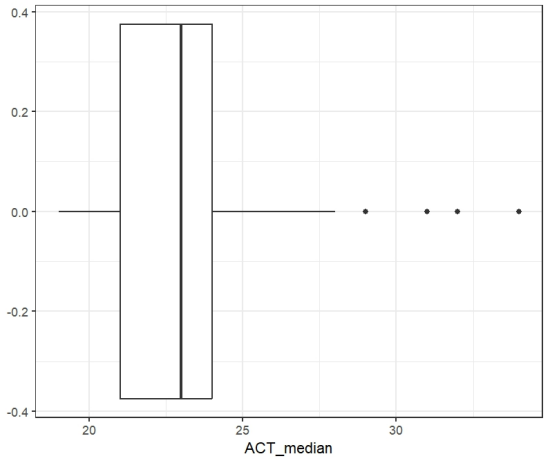
Extra on Outliers

Outlier in a scatterplot

- very small or large values for one of the variables (or both!)
- does not follow the overall pattern







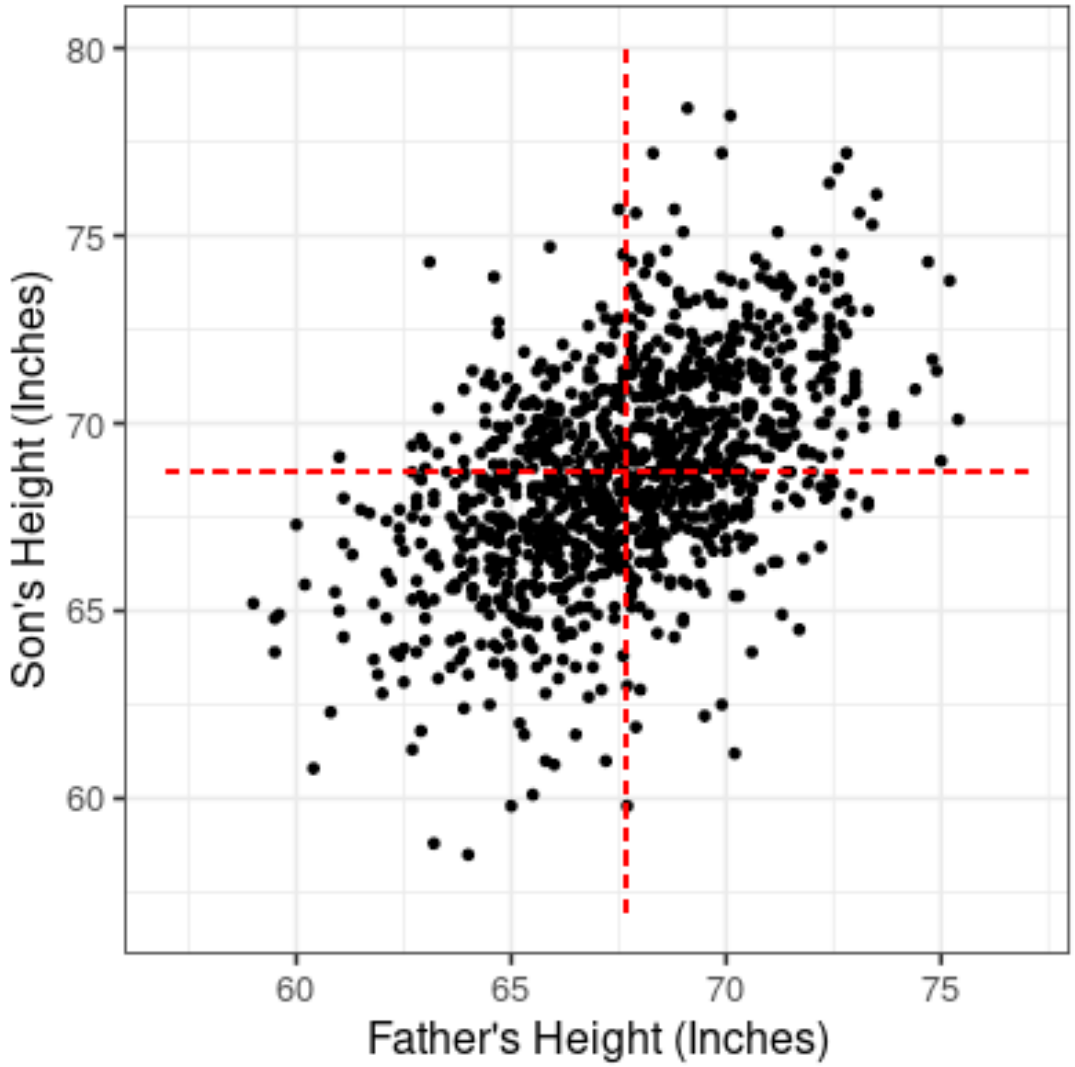
Pearson's Height Data

In the 1880's the Western scientific community was enthralled with the idea of quantifying heritable traits

Karl Pearson collected data on the heights of 1,087 father's and their fully grown first born sons

Father	Son
65.0	59.8
63.3	63.2
65.0	63.3
65.8	62.8
61.1	64.3
63.0	64.2
⋮	⋮

Height Data


Son's Height (Inches)
80
75
70
65
60
60
65
70
75
Father's Height (Inches)

Pearson's Correlation Coefficient

Heights clearly associated, but how to quantify?

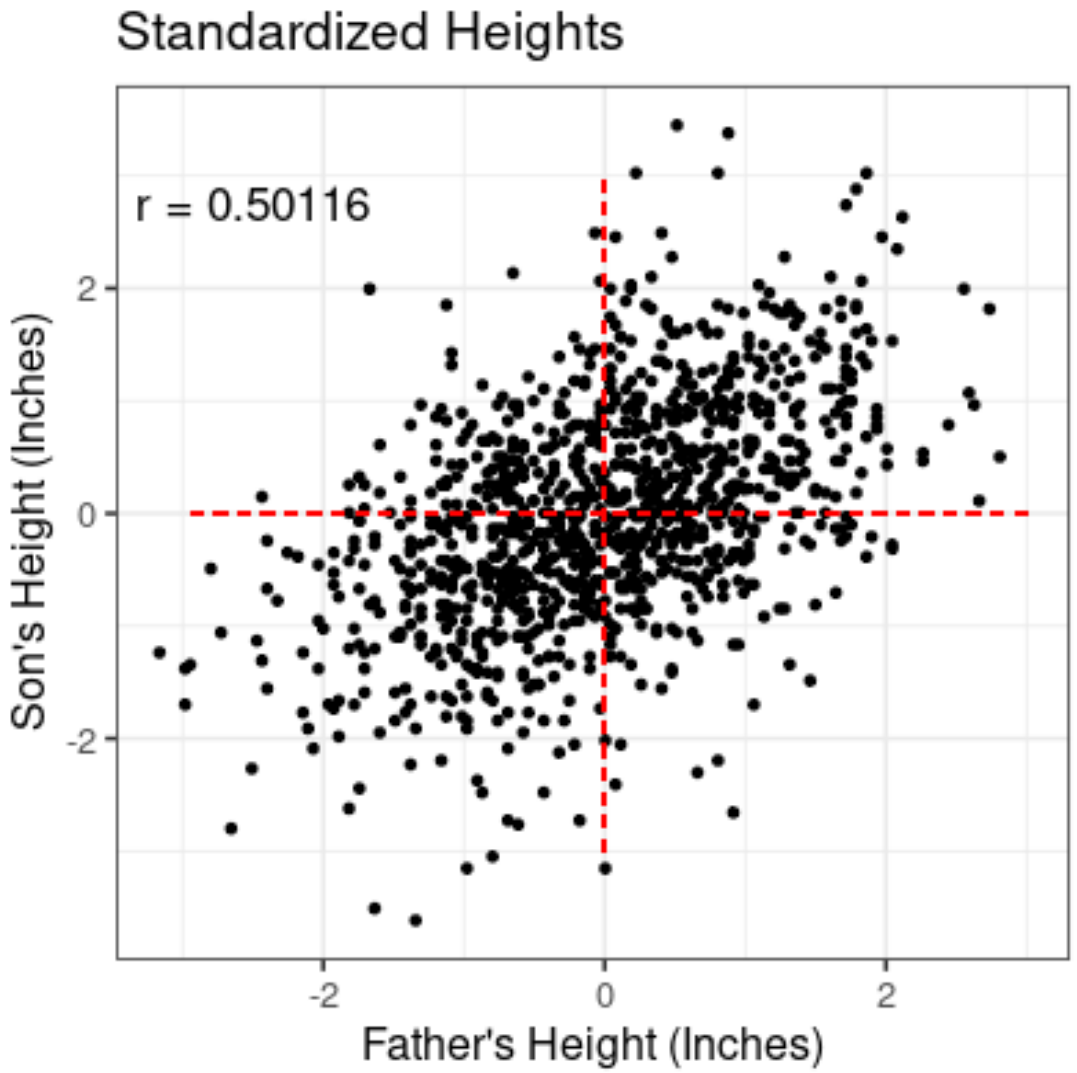
Building upon the work from French scientist Francis Galton, Pearson developed the **Pearson's correlation coefficient (r)**:

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i - \overline{x}}{s_x}\right)\left(\frac{y_i - \overline{y}}{s_y}\right)$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}(z_{x_i})(z_{y_i})$$

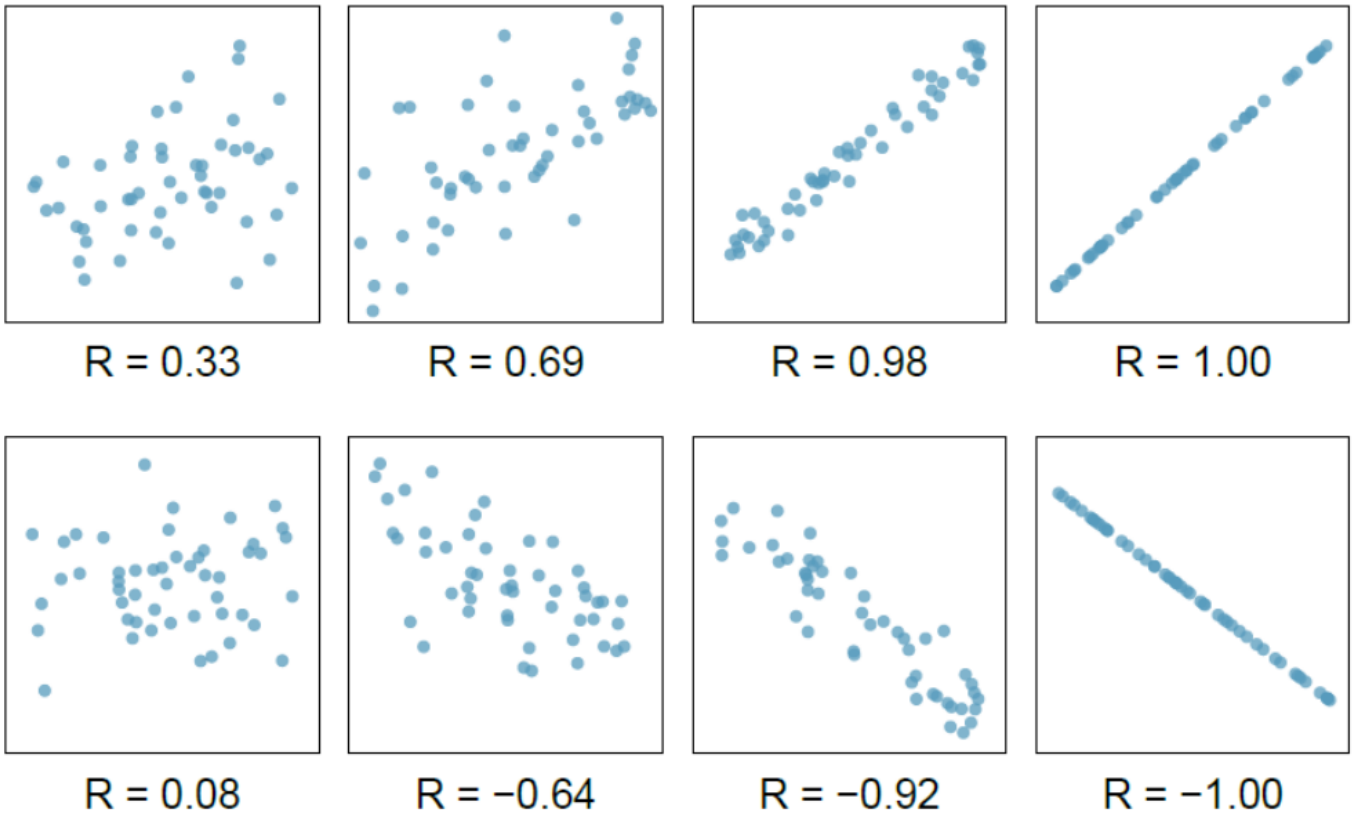
If above-average values of X are common among cases with above-average values of Y (or vice-versa), we should expect r to be positive

Height Data – Standardized



Correlation Examples

Pearson's correlation coefficient tells us the strength of *linear* association between two quantitative variables (and direction!)

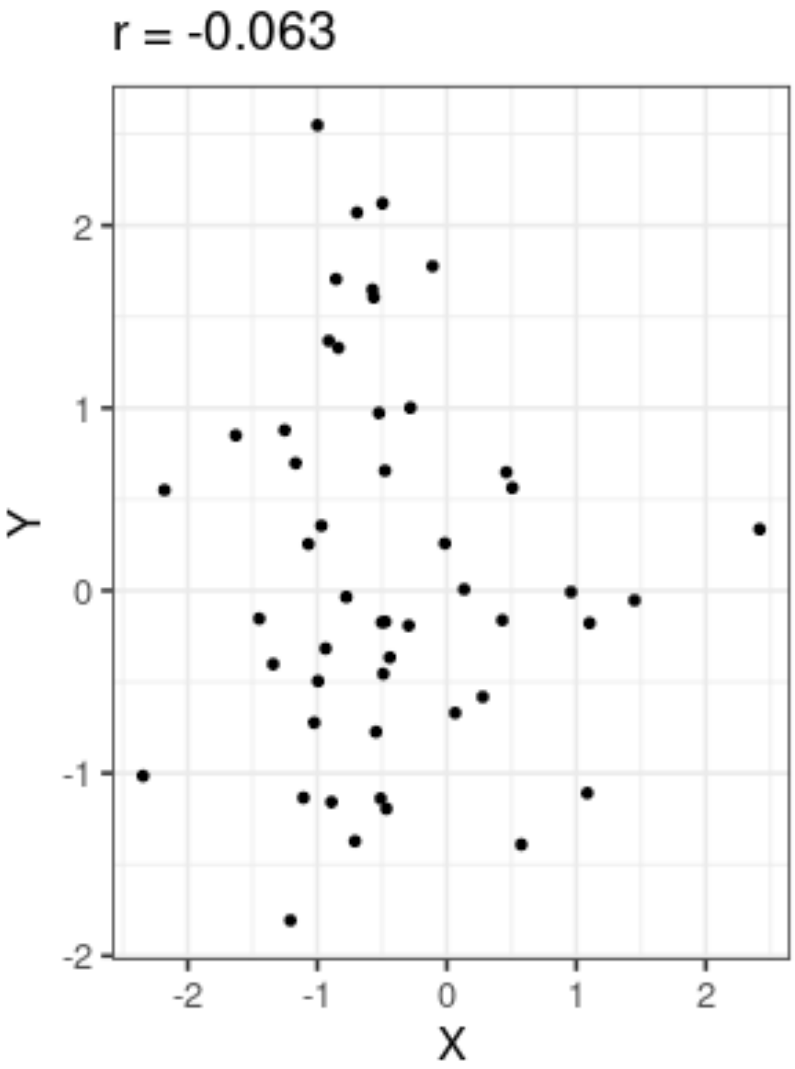


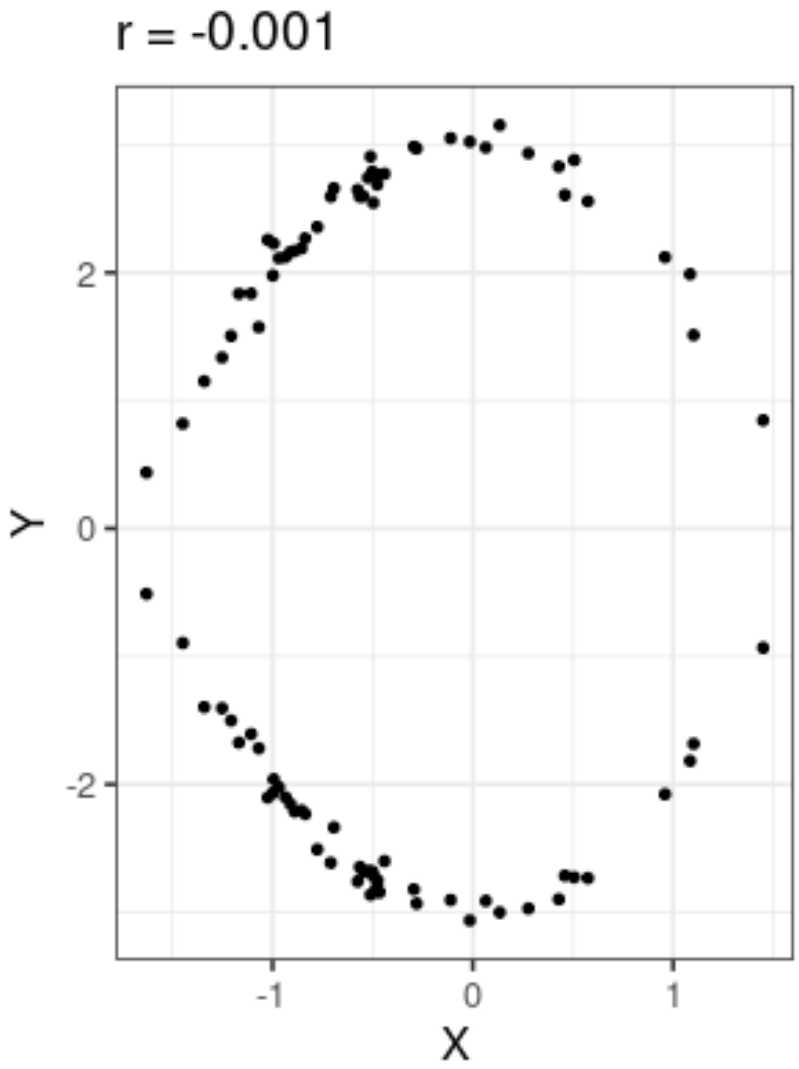
What is considered "strong"?

Correlation Coefficient		Dancey & Reidy (Psychology)	Quinnipiac University (Politics)	Chan YH (Medicine)
+1	−1	Perfect	Perfect	Perfect
+0.9	−0.9	Strong	Very Strong	Very Strong
+0.8	−0.8	Strong	Very Strong	Very Strong
+0.7	−0.7	Strong	Very Strong	Moderate
+0.6	−0.6	Moderate	Strong	Moderate
+0.5	−0.5	Moderate	Strong	Fair
+0.4	−0.4	Moderate	Strong	Fair
+0.3	−0.3	Weak	Moderate	Fair
+0.2	−0.2	Weak	Weak	Poor
+0.1	−0.1	Weak	Negligible	Poor
0	0	Zero	None	None

Source: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6107969/

Correlation Examples





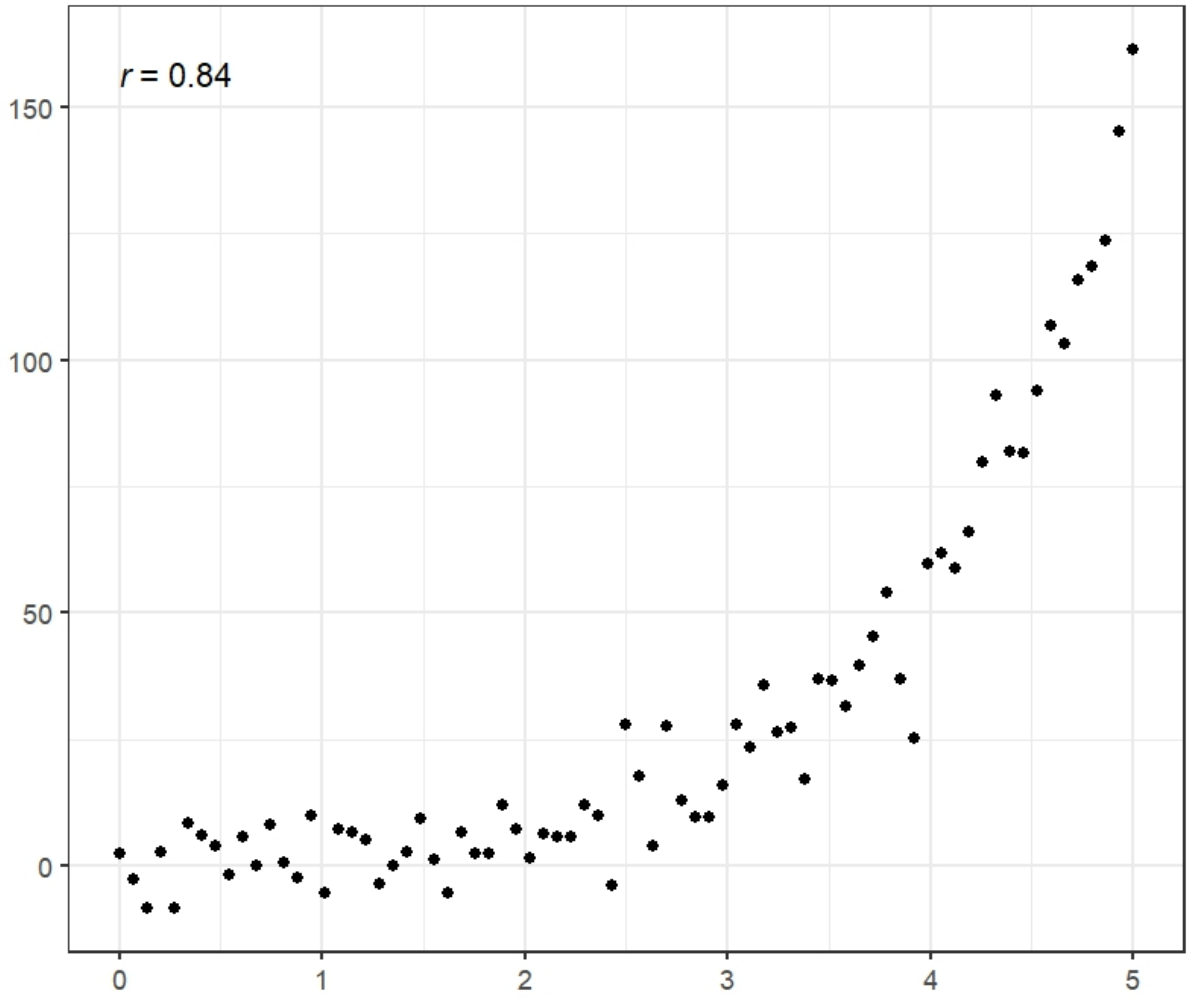
Correlation Properties

Properties:

- r has no units/is unitless
  - changing scale of either variable doesn't affect r value
- r measures the strength of a LINEAR relationship
  - Gives an idea of how well the scatterplot follows a straight line
  - Curved lines can be 0, regardless of "strength"
- r is between -1 and 1
- The closer r is to 0 $\rightarrow$ weaker linear relationship
- The closer r is to 1 or -1 $\rightarrow$ stronger linear relationship
- r=0 $\rightarrow$ no linear relationship

Pitfalls

If we get a value for r close to +1 or -1, it does **not** mean the relationship actually is linear (double-check the scatterplot!)



Pro tip: You should graph before you see the numeric summaries, and as

Non-linear Association

In addition to Pearson, we have **Spearman's rank correlation** (denoted ρ) where the values of X and Y are replaced with their rank order from smallest to largest before correlating:

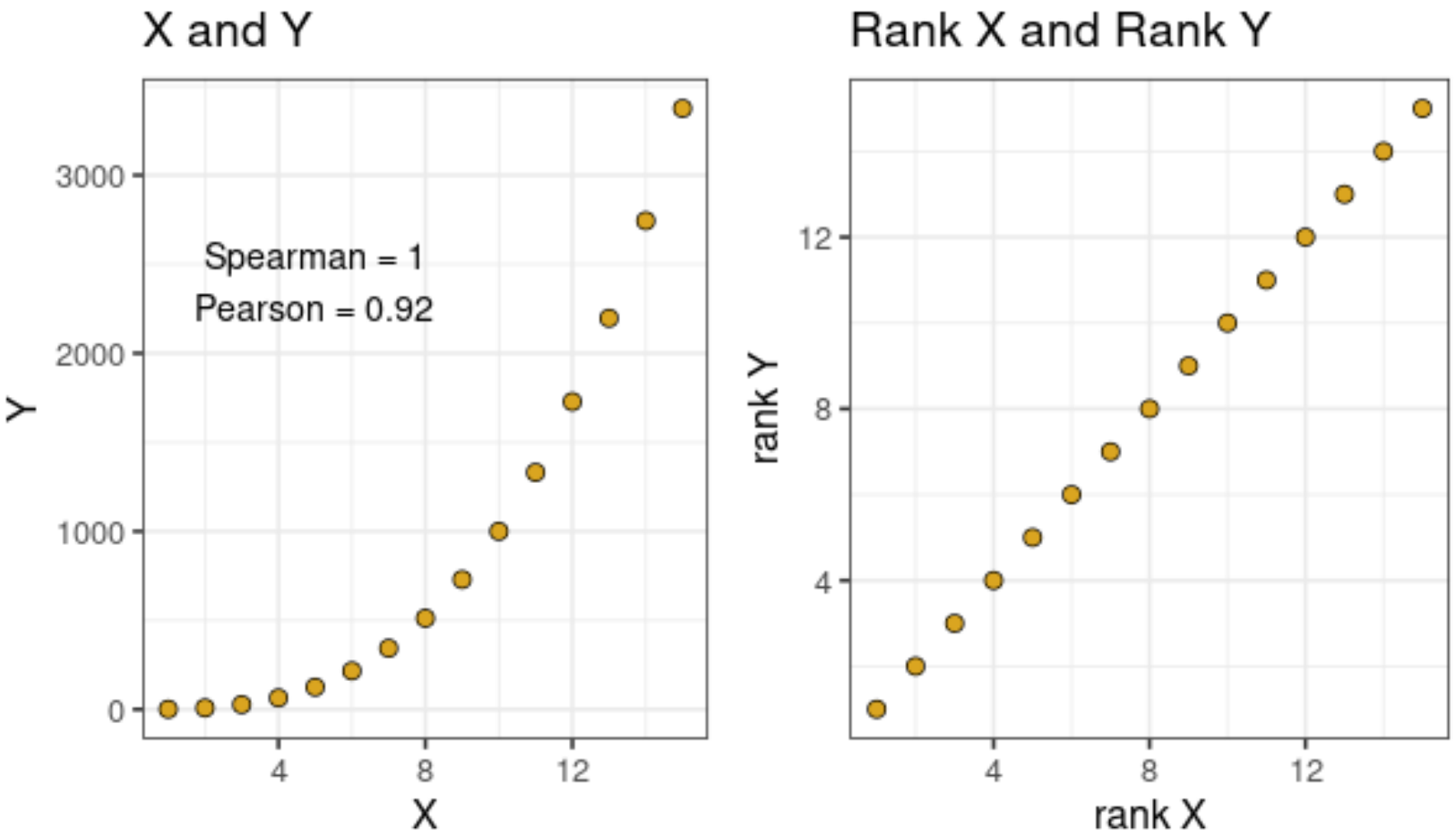
$$\begin{aligned} X &= \{2, 4, 6, 9, 8\} \\ Y &= \{7, 4, 1, 5, 3\} \end{aligned} \quad \Longrightarrow \quad \begin{aligned} X_{rank} &= \{1, 2, 3, 5, 4\} \\ Y_{rank} &= \{5, 3, 1, 4, 2\} \end{aligned}$$

Whereas Pearson's r measures *linear association*, Spearman's ρ measures the *monotonic association* (increasing or decreasing)

- Can think of this as a non-parameteric version of Pearson's correlation
- Also can be applied to ordinal data

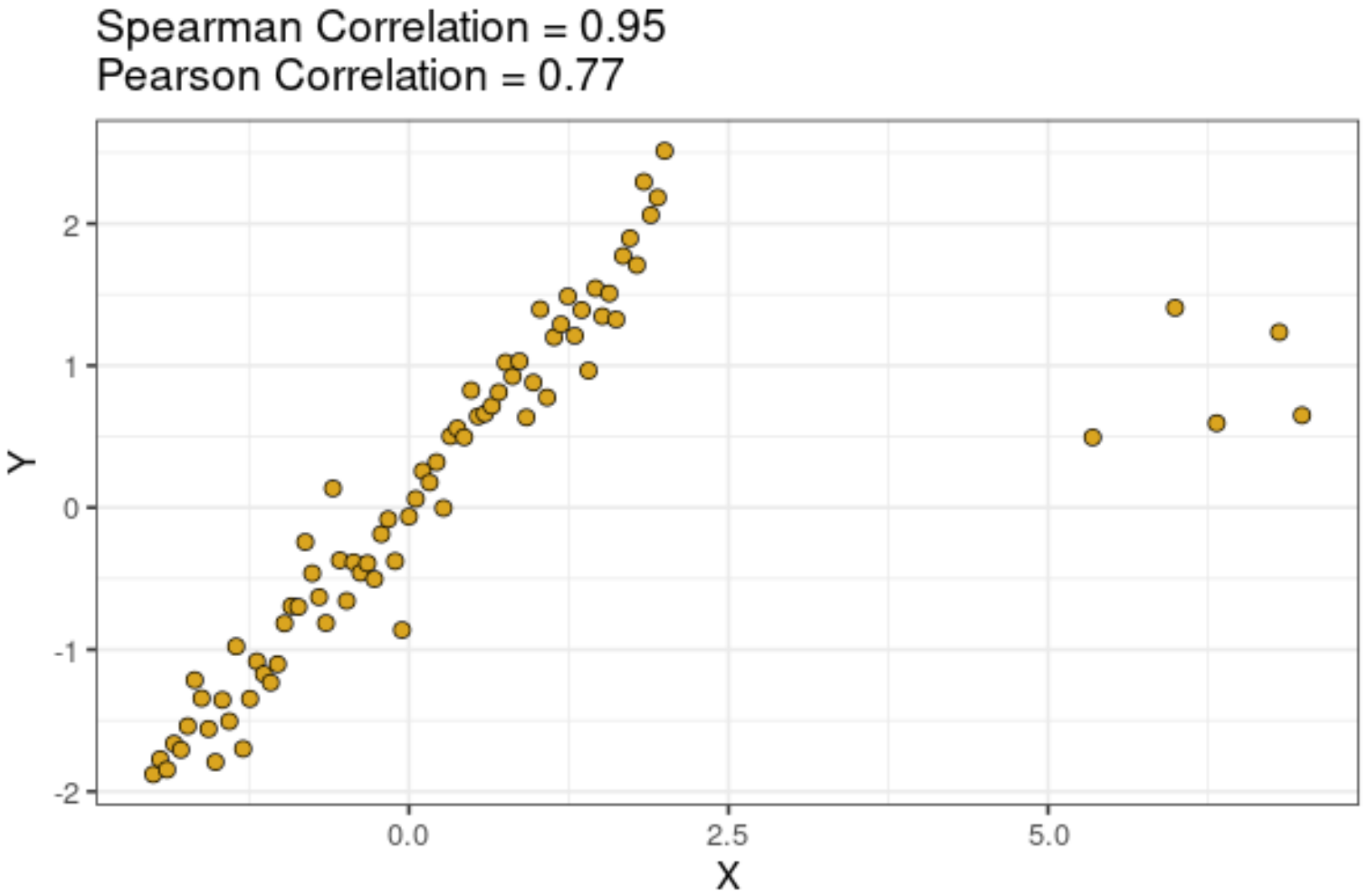
Non-linear Assocation

$$y = x^3$$



Spearman Correlation

Spearman's correlation is more robust to outliers



Question

Q: If data sets have the same (or similar) summary statistics do they act the same?

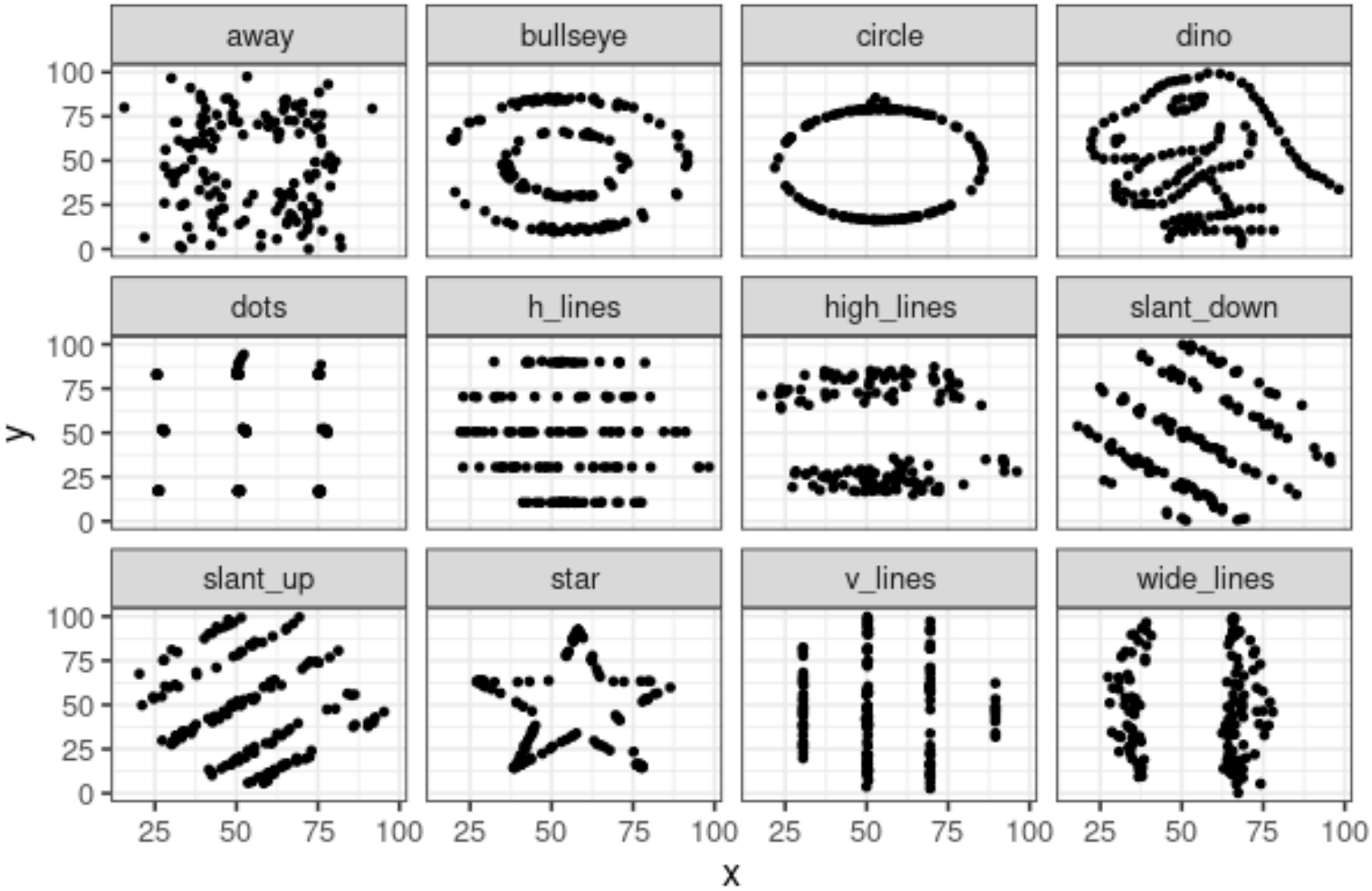
Answer

Q: If data sets have the same (or similar) summary statistics do they act/look the same?

No!

- Summary statistics are summaries only
  - eg heavily processed data
  - 7 statistics are an extreme reduction from, say, 400 observations
- Graphing is the common way to decide that two data sets are similar
- ?anscombe in R for Anscombe's Quartet data setS

"Datasauraus Dozen"



Ecological Correlation

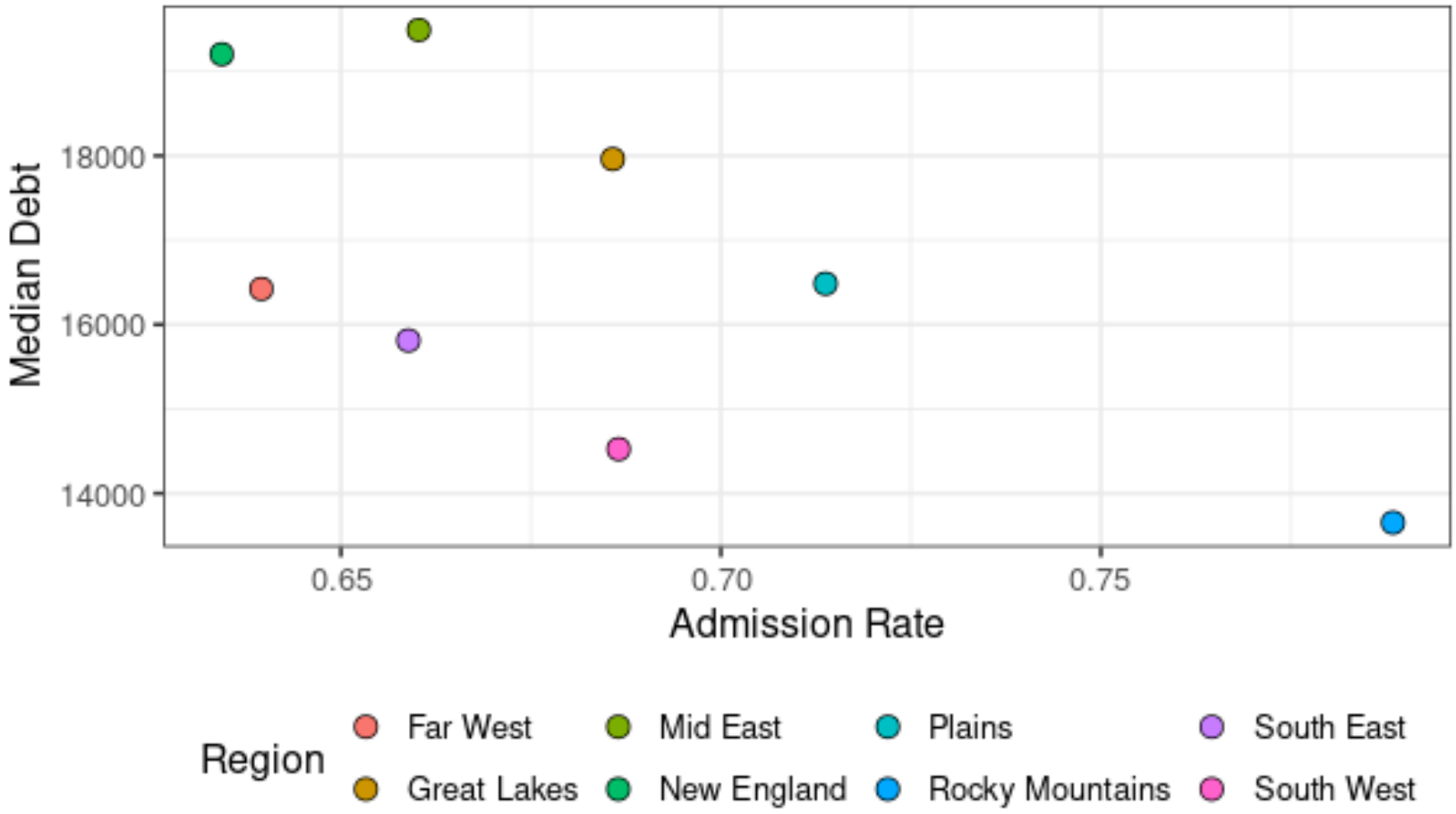
Ecological correlations compare variables for data that have been aggregated at an ecological level

- Countries
- States
- Schools

The *ecological fallacy* is a fallacy in which a conclusion is drawn that, because a correlation exists at a group level, it must exist at the individual level as well

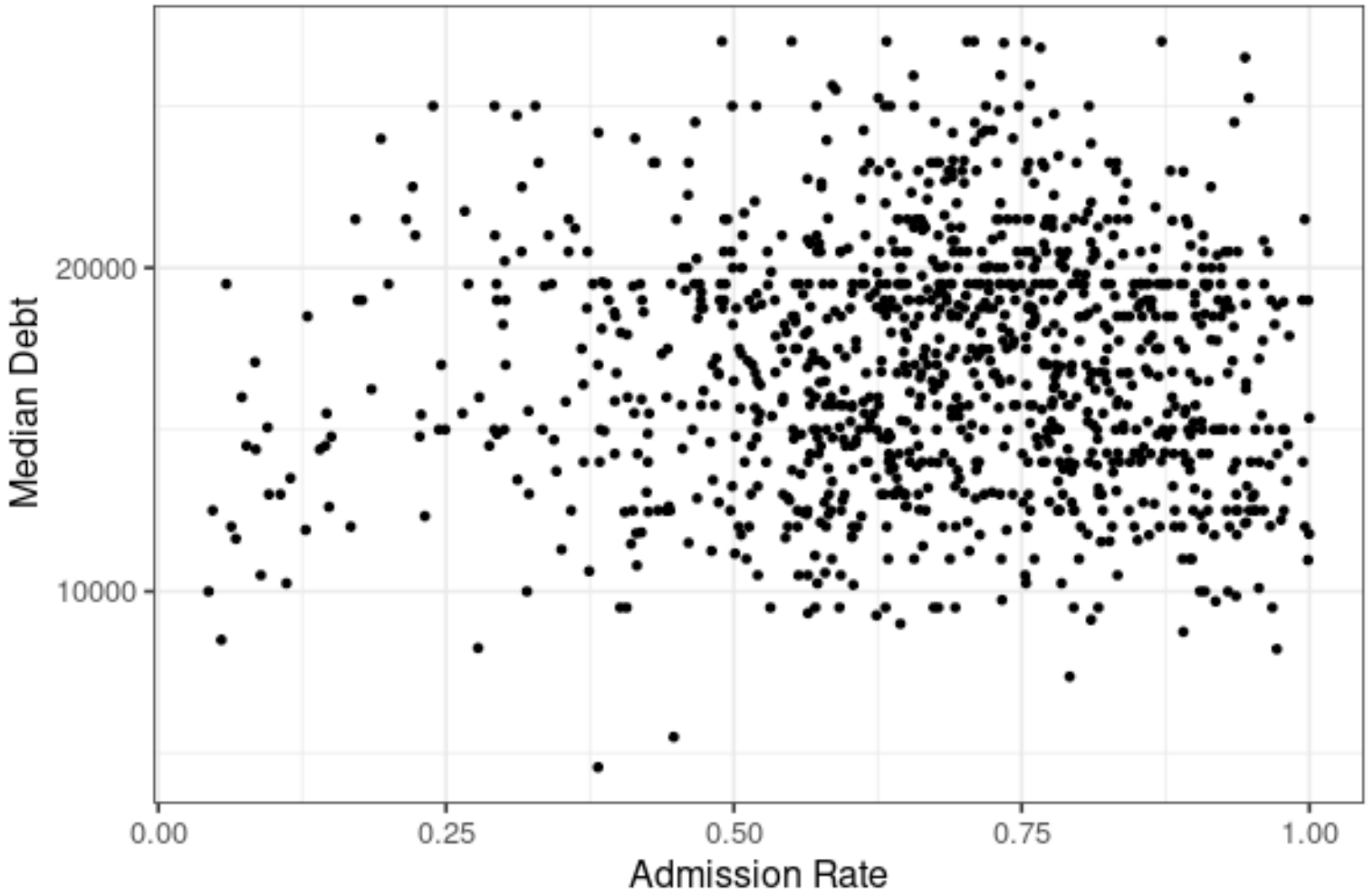
College Ecological Fallacy

Grouping by region, the correlation between (mean) admission rate and (mean) median debt is $r = -0.66$

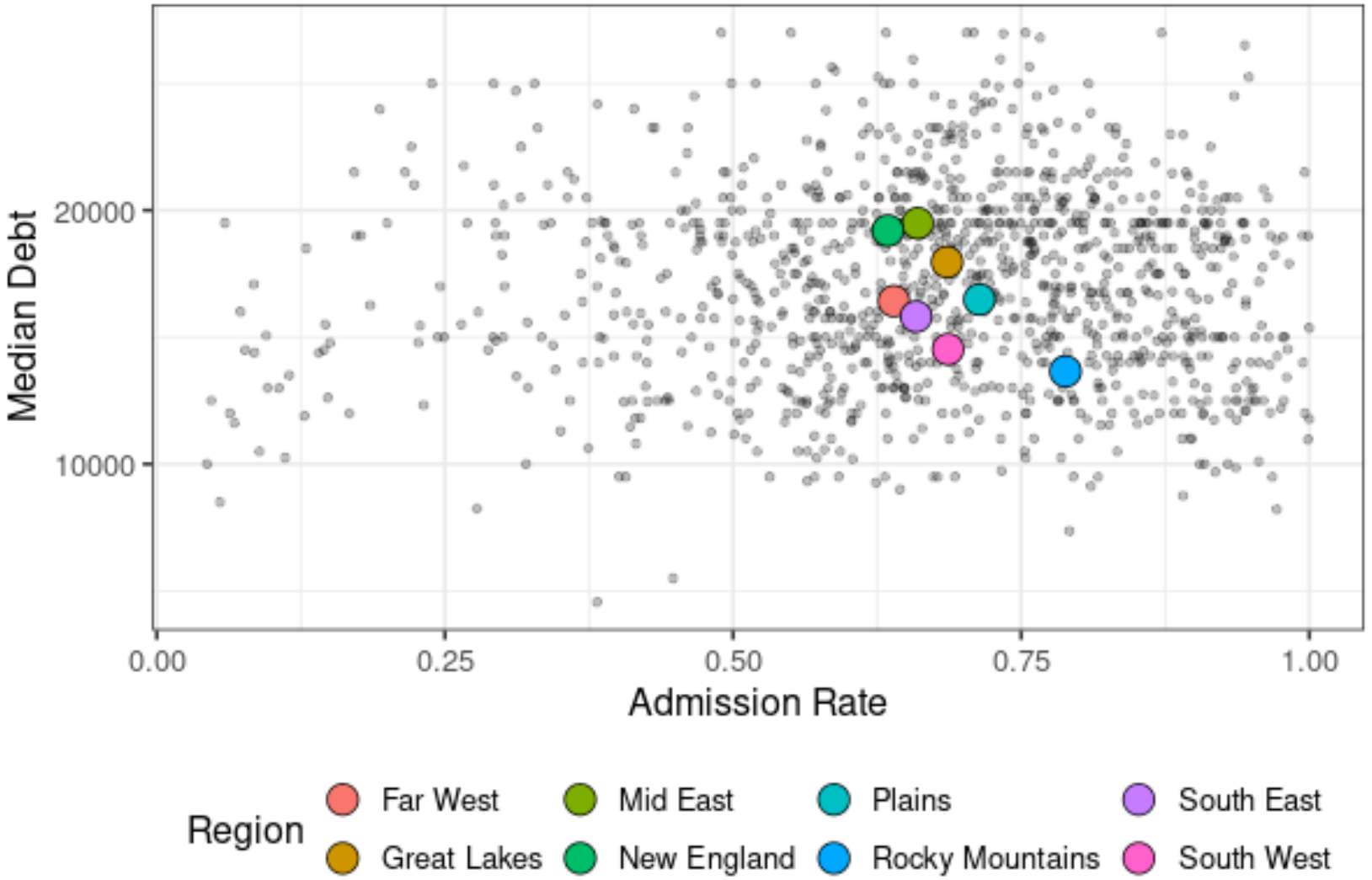


College Ecological Fallacy

This completely disappears when we remove consideration of region, with $r = 0.02$



College Ecological Fallacy



Correlation $\neq$ Causation

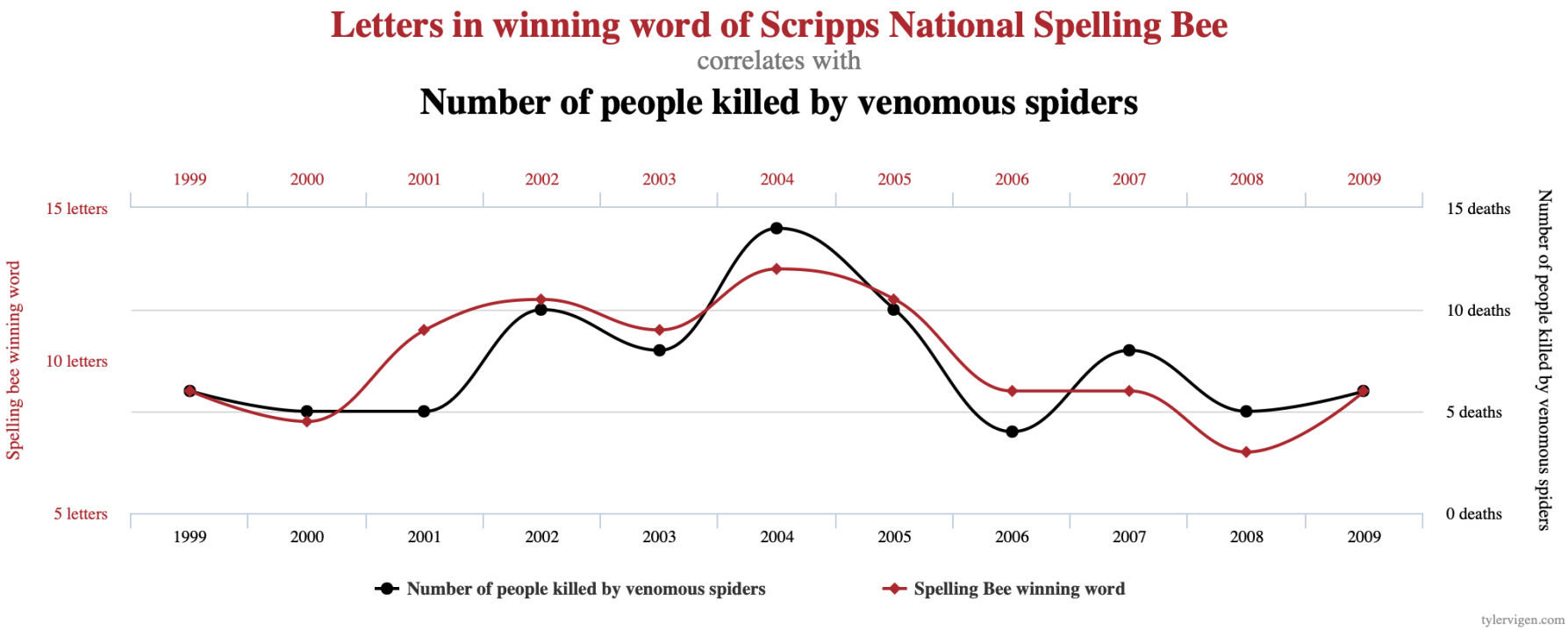
We can have a large correlation value between 2 variables. This does not mean the explanatory variable is *causing* a change in the response variable.

Examples with high correlation but where no causal claims can be made:

- Literacy Rate and Gross Domestic Product (GDP) in countries
- average number of TVs in a household and Life expectancy of countries
- ice-cream sales and shark attacks

Lurking Variable: a third variable that explains the relationship between two variables with high correlation

Correlation



Review

- **Pearson's correlation** strength of *linear association* (and direction)
- **Spearman rank correlation** useful for data with outlier's or non-linear (but monotone) relationship
- Be careful with **ecological correlations** – inference for a group is not always valid for individuals
- Correlation $\neq$ Causation