Numerical Summaries

and Boxplots

Grinnell College

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Unit 1

1. **Data Visualization** displaying data to allow quick and easy interpretations
   - The best graphics tell a story

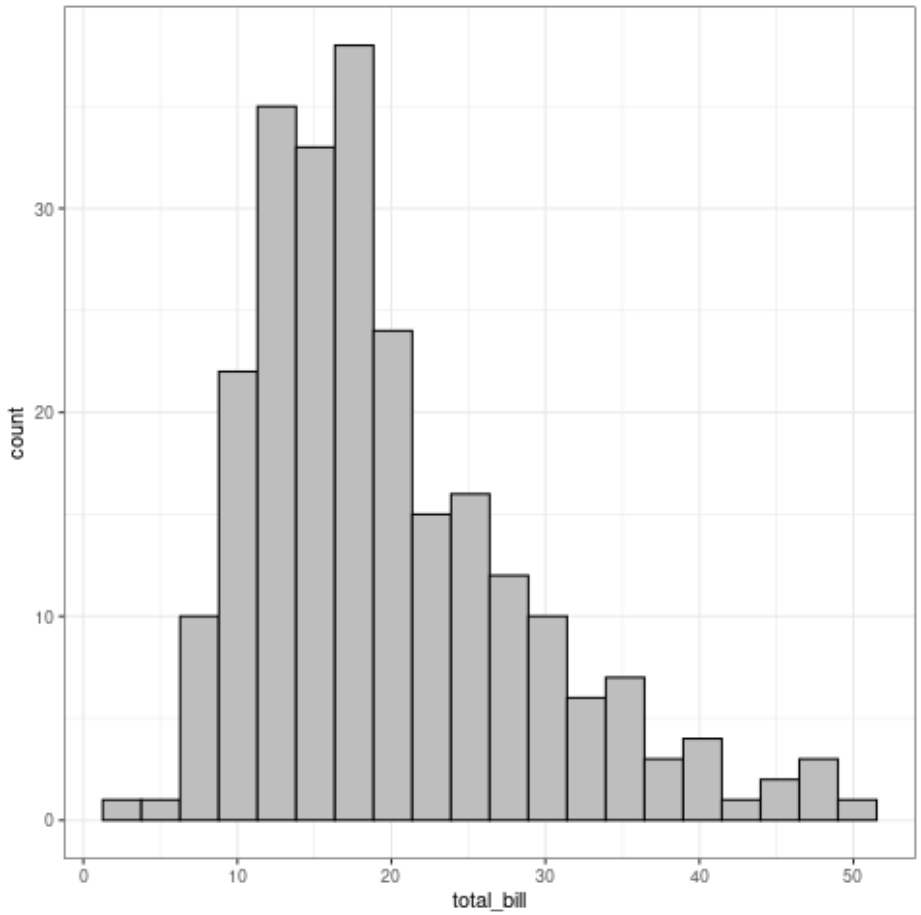
2. **Numerical Summaries** that tell us about the data
   - Summaries necessarily lose information
   - Have already touched on many of them

3. **Linear Regression** which allows us to describe the relationship between variables with more nuances
   - Explains variability we are seeing
   - Makes predictions based on our sample's data

Today we'll hit numeric summaries and wrap up graphics.

Review – One Quantitative Variable

We've seen a few examples of histograms at this point.



Things we discuss when talking about a distribution?

Review – Quantitative Distribution

We need to mention all of the following things when we describe the **distribution** of a quantitative variable:

- **Shape** - how does the distribution look?
  - - symmetric?
  - - skewed?
  - - # of modes
- **Outliers** - are there values that are much smaller/larger than the rest?
- **Center** - where does the data bunch up
- **Spread** - how spread out is the data

Today will tackle the last two points.

Two Approaches for Center and Spread

1. Order Statistics
   - Based on percentiles
     - ★ Median, IQR, Range
     - ★ Min, Q_1, Median, Q_3, Max
   - Called "order" because of ordering the values smallest to largest
   - Non-parametric statistics/applied to any shape

2. Moment Statistics
   - Based on mathematical properties of our data
   - Eg Mean and Standard Deviation
     - ★ Less common eg: skew, kurtosis, rate parameters
   - Makes assumptions on the shape of the distribution
     - ★ Only as valid as those assumptions!!!
   - Have nice properties usually
     - ★ Eg can use calculus

Order Statistics

Order statistics are numerical summaries based on the ordered ranking of a quantitative variable (smallest to largest)

There are a few properties in particular that make order statistics useful:

1. They make no assumptions about how the data is distributed
2. Are generally *robust* to major fluctuations in the data
   - Eg Outliers do little to them
3. Easier to interpret

Percentiles

A **percentile** α is a number such that $\alpha\%$ of our (quantitative) observations fall at or below this number when ranked from smallest to largest

Some percentiles have special names. The *median*, for example, is the 50th percentile.

Other notable percentiles include:

1. Minimum
2. 25th percentile or **first quartile** (Q_1)
3. 75th percentile or **third quartile** (Q_3)
4. Maximum

Median (Center)

The **median** is another name for the 50th percentile. It is frequently used as a measure of center.

These are some other ways to think about the **median**.

- the **median** divides the data into an upper and lower half
- the middle value of the data if arranged from smallest to largest
- about half the data is larger than the median and about half the data is smaller

Quartiles and IQR and Range

Quartiles are the names for the 25% of data markers

- $Q_1 = 25\%$
  - = median of the lower half of the data
- $Q_3 = 75\%$
  - = median of the upper half of the data
- If the number of observations is odd, include the median when finding the median of the lower and upper half
  - Eg 1,1,2,3,5 - (1,1,2) and (2,3,5)

IQR is the width of the middle 50% of the data

- Q_3 - Q_1

Range is the total width of your data

- Max - Min
- Largest variability seen in the data

Question?

What's Q_2?

Five Number Summary

The five number summary is made up of...

1. Minimum
2. Q_1
3. Median
4. Q_3
5. Maximum

These make no assumptions!!

Five Number Summary

Fibonacci Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34

Find the 5 number summary of the above

Five Number Summary

Fibonacci Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

Find the 5 number summary of the above

1. Minimum: 1
2. Q_1: 2
3. Median: $13/2 = 6.5$
4. Q_3: 21
5. Maximum: 55

5 Number Summary vs Median and IQR

I generally prefer the 5 number summary, why?

5 Number Summary vs Median and IQR?

I generally prefer the 5 number summary, why?

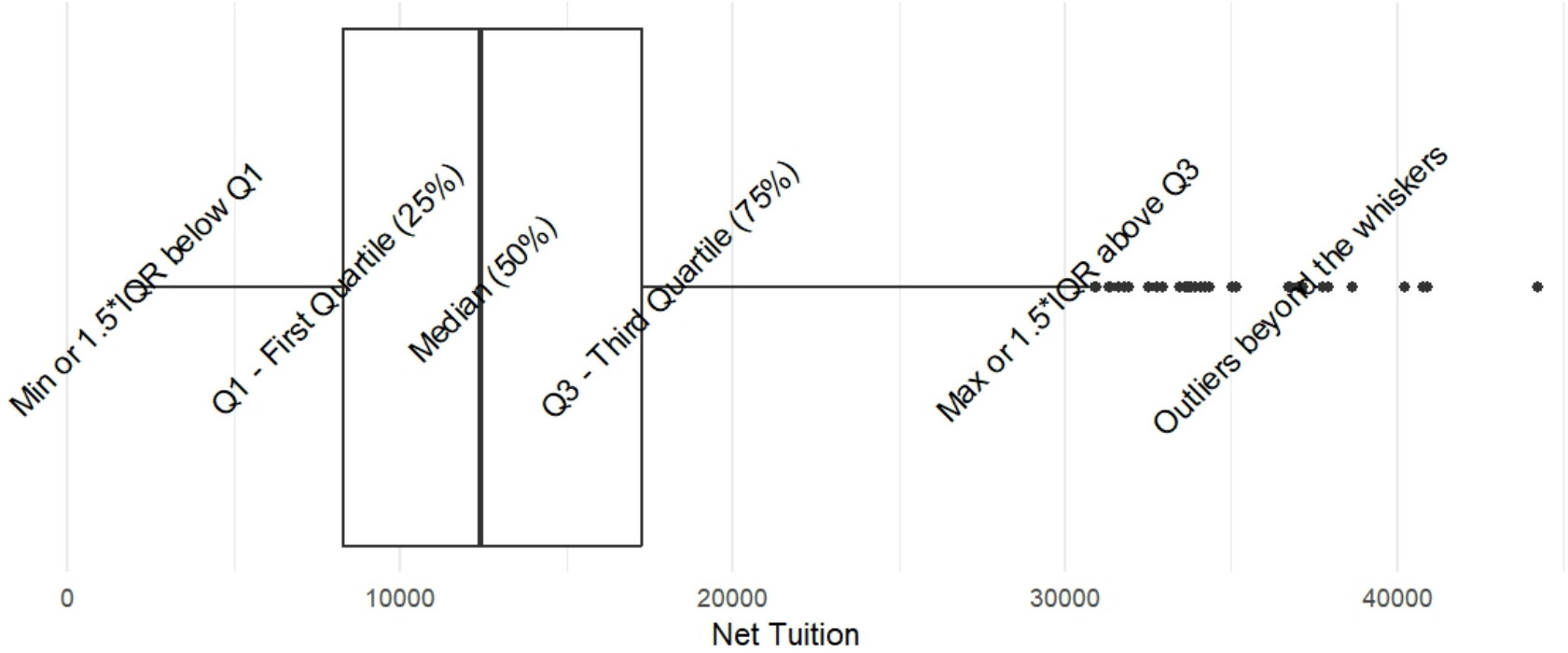
The data is less *processed*

- Information is lost when we summarize
- I can find IQR with the 5 number summary
- I can't find 5 number summary with median and IQR

Boxplots

A boxplot is another way to display a quantitative variable as a rival to a histogram, specifically it displays the 5-number-summary

ex) 2019 College data



Boxplots

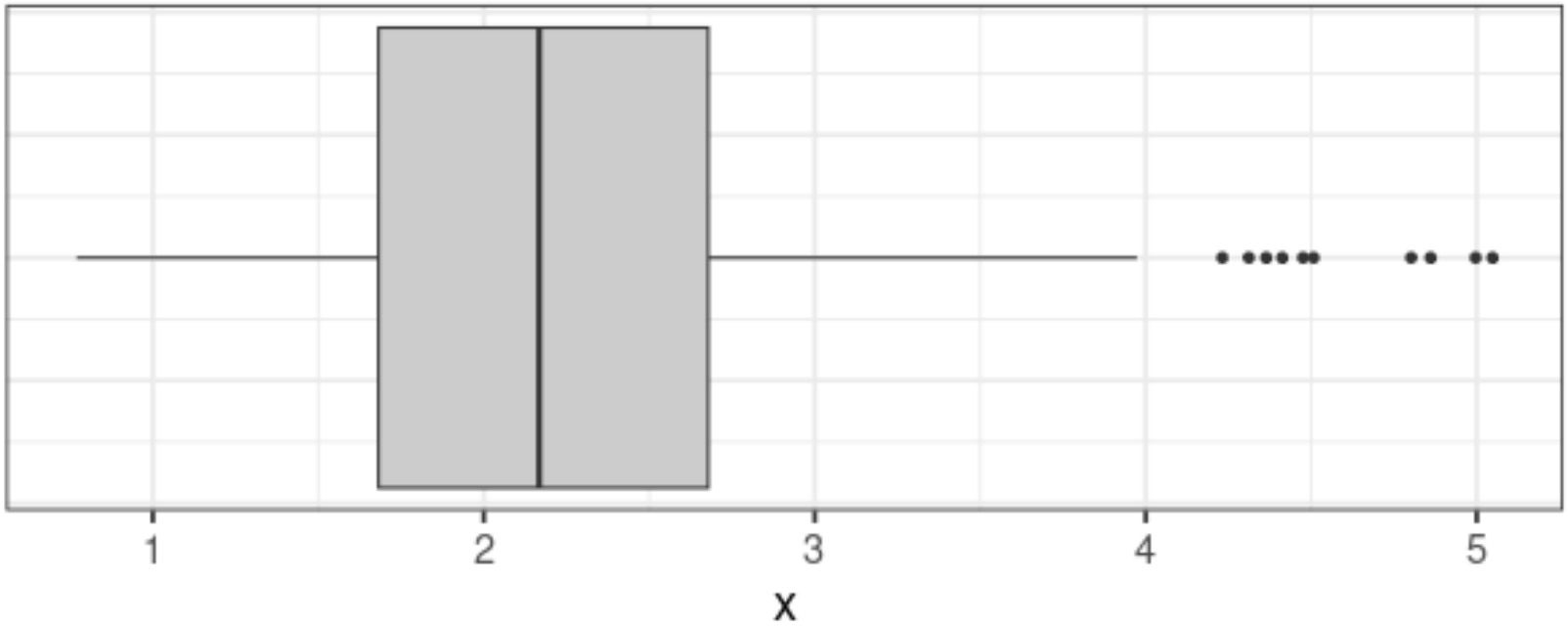
The two parts are...

1. The box...
   - Q_1
   - Median
   - Q_3

2. and whiskers
   - Q_1 - 1.5 IQR
   - Q_3 + 1.5 IQR
   - MUST BE PLACED ON A POINT!!
     - ★ Boxplots are all about ordered statistics which are observed quantities
     - ★ So pull back the lines towards the median
     - ★ Don't just let them end where you don't have an observation

Boxplots

Min, Q_1, Median, Q_3, Max

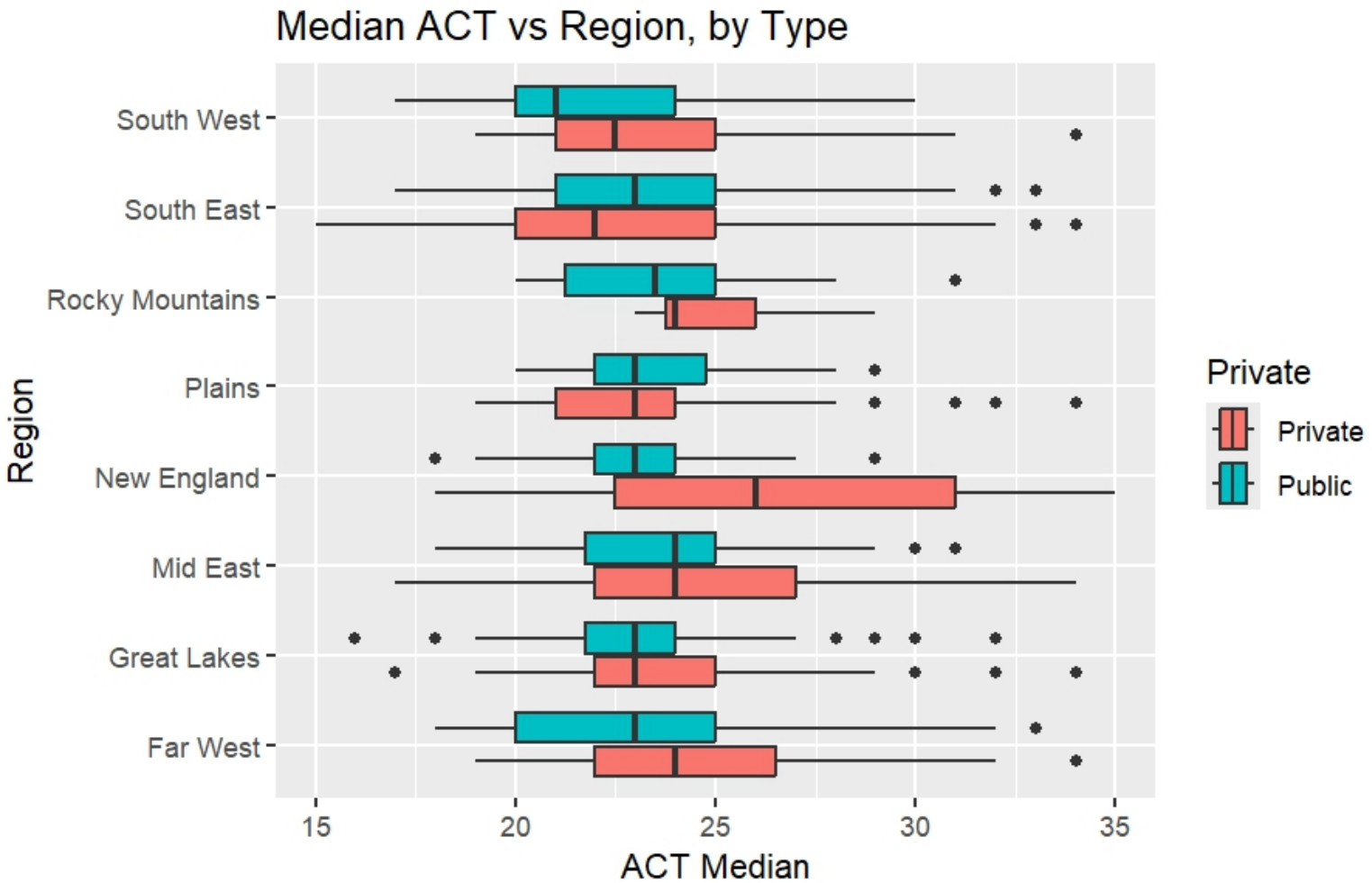


Challenge: Visually guesstimate the five number summary of this boxplot

("Guesstimate" is a highly technical term.....)

Boxplots

Boxplots shine when there are too many categories for histograms to plot



Moment Statistics

Moment statistics are statistics that are based on specific mathematical properties of our data (mean and st. dev.).

- Often have very nice properties
  - eg we can use calculus
- Make assumptions about the shape of the data
  - Assumptions bring information a priori but if violated leads to problems (bias)
- Can be moderately sensitive to fluctuations in the data
  - ie NOT robust
  - eg outliers wreck the mean

Notation

Let's get on the same page for notation...

- Sample size is denoted with **n** (or N)
- $\mathbf{x}_i$ is used to denote the i^{th} value of x in our data set
  - eg $\mathbf{x}_3$ is the 3rd value of x
- A bar over a variable indicates an estimated mean
  - $\overline{x}$ is the mean of x in our sample
- $\sum$ stands for "the summation of" (ie add these up)
  - $\sum_{i=1}^{5} (i) = 1 + 2 + 3 + 4 + 5 = 15$
- Not needed but $\prod$ is for the product of things

Mean (Center)

In common parlance the (arithmetic) **mean** is the same thing as the **average**.

- To find the value of the mean, we add up all the values of the variable and divide by the number of observations.
- Often the *sample mean* of the variable x is denoted as $\overline{x}$
- Using the notation from the previous slide, the equation for the mean is

$$\overline{x} = \frac{\sum x_i}{n}$$

Spread – Standard Deviation

Standard Deviation is the measure of typical deviation (distance) of all the observations from the mean

- **s** is often used to denote the standard deviation of our sample

$$s = \sqrt{\frac{1}{n-1}(x_i - \bar{x})^2}$$

Why do we take the square root? Taking the square root ensures that the standard deviation has the same units as the original variable

Why do we use n-1 and not n? It's complicated. Using n-1 gives us unbiased estimates and predictions. Vaguely it's the price paid for using the mean in our formula

Standard Deviation

Some properties of the **standard deviation**:

- measures spread (variability) from the mean
  - values close to the mean = smaller contribution to s
  - values far away from the mean = larger contribution to s
- cannot be negative ($s \geq 0$)
- has the same units as the original variable

You may hear the word **variance**.

- sample variance = s^2
- harder to interpret
- Many distributions are defined with variance instead of s
  - Eg Wiki for Chi-squared Distribution only has variance listed
  - Higher up in statistics "variance" is more prevalent

Which measures to use?

The shape of the distribution, as well as whether we have outliers will determine whether we use order statistics (median, IQR, and range) or moment statistics (mean and standard deviation) to describe the center and spread

- Generally default towards moment statistics
- Up to the histogram to convince me to use order statistics
  - For those who know the term, order statistics lack the power of moment statistics
  - But they (order stats) also don't have assumptions

Which measures to use?

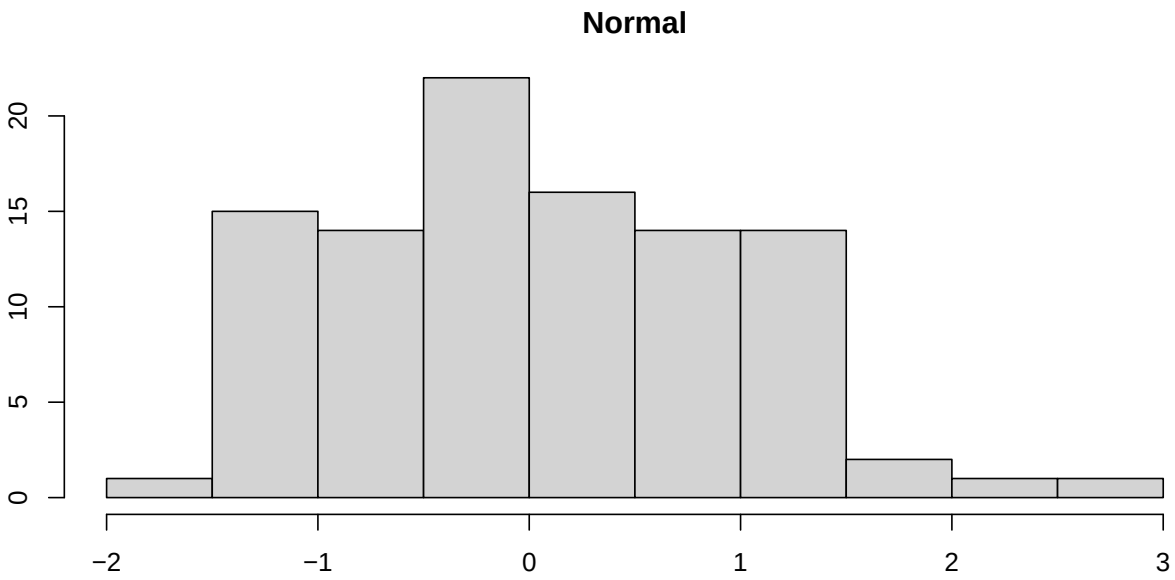
Order statistics are robust, moment statistics are not.

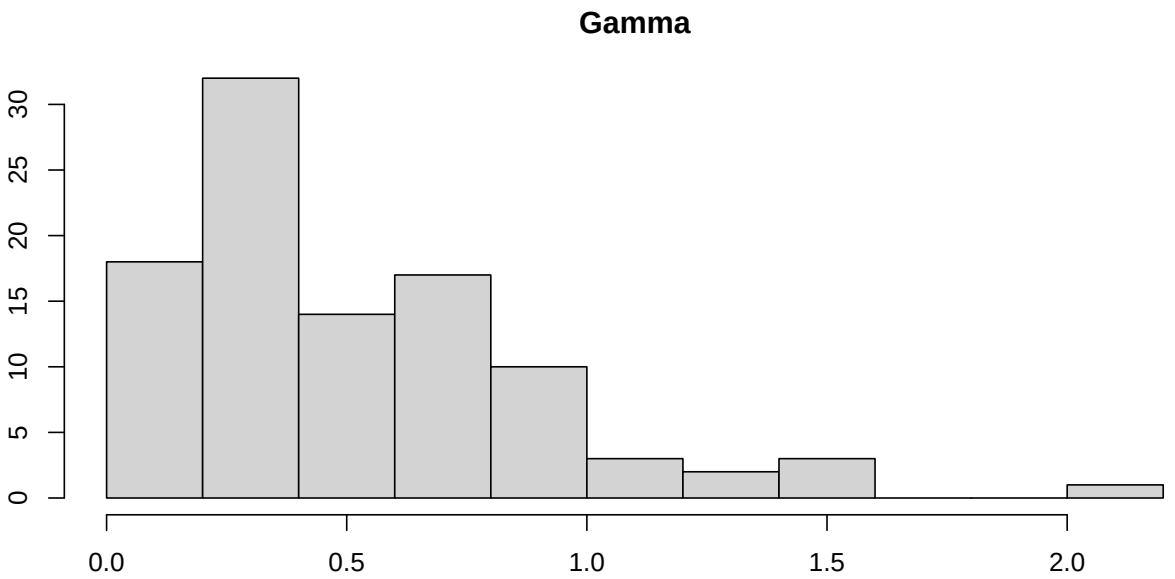
- A skewed distribution can affect the mean and std. dev. a lot
  - skew $\rightarrow$ mean & std. dev. not good measures of center & spread
- Outliers can affect the mean and std. dev. a lot
  - outliers $\rightarrow$ mean & std. dev. not good measures of center & spread

Summary:

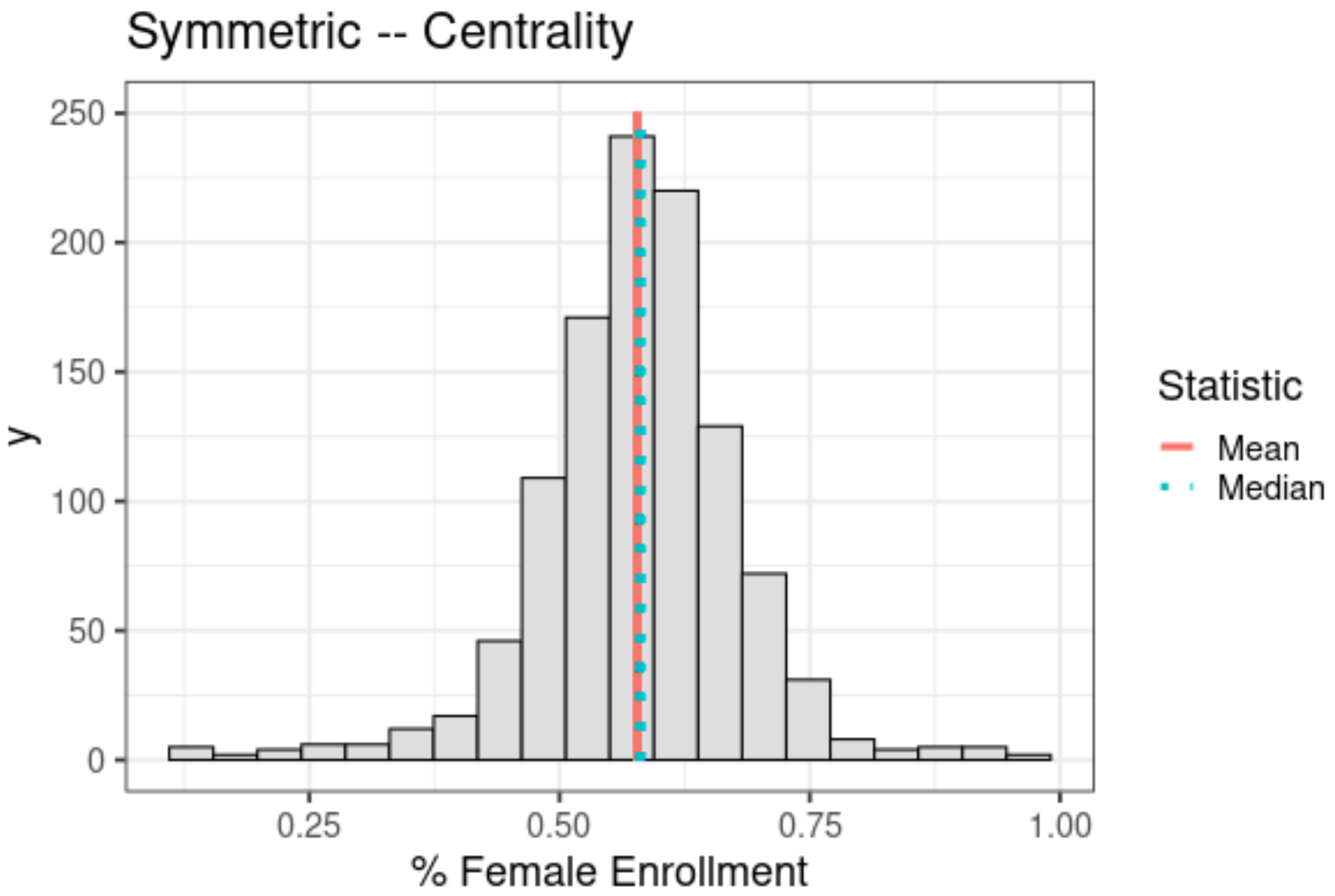
Symmetric shape with no 'extreme' outliers $\rightarrow$ mean and std. dev.
Skewed shape or outliers (or both) $\rightarrow$ median and IQR

Eg

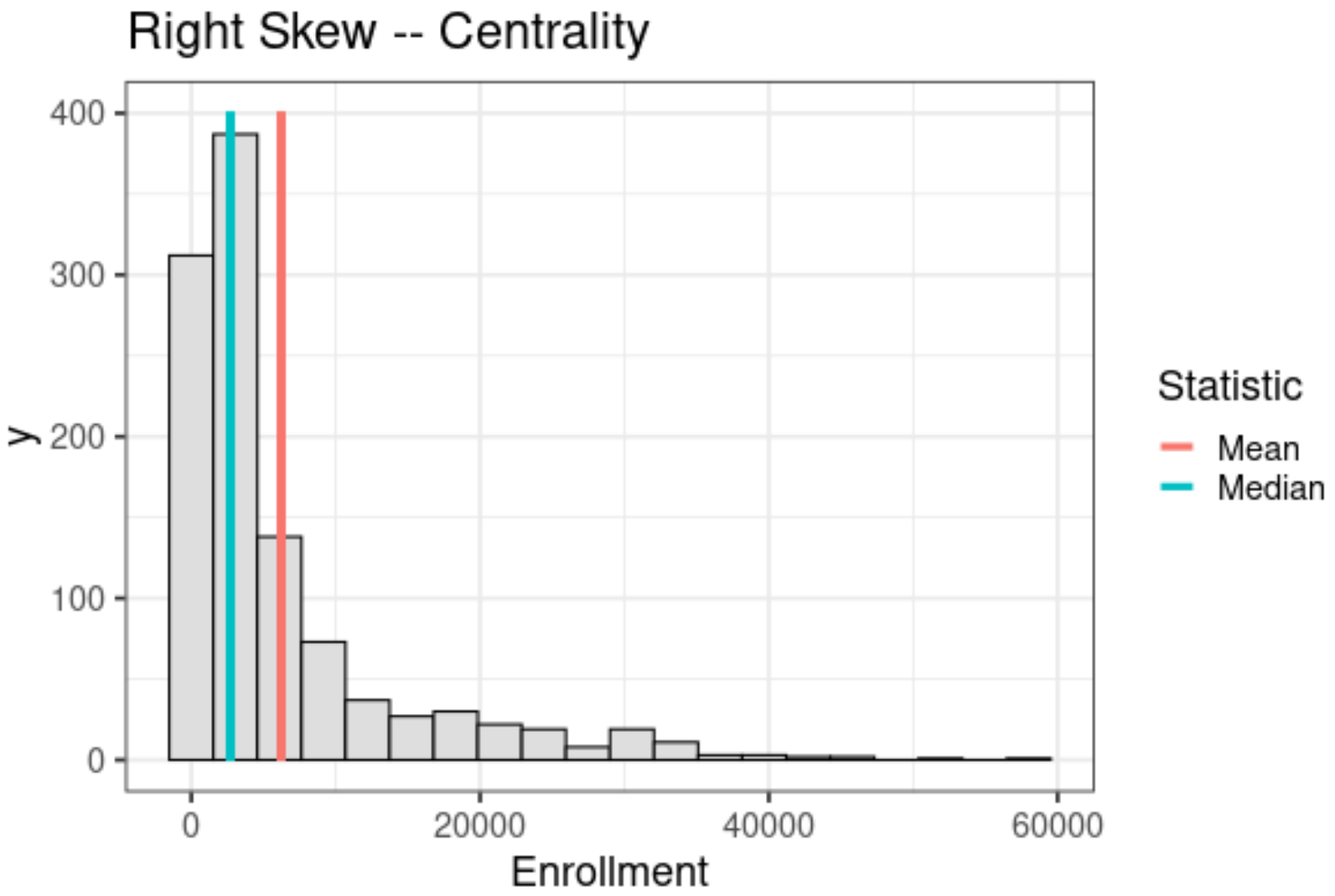




Comparing Mean with Median



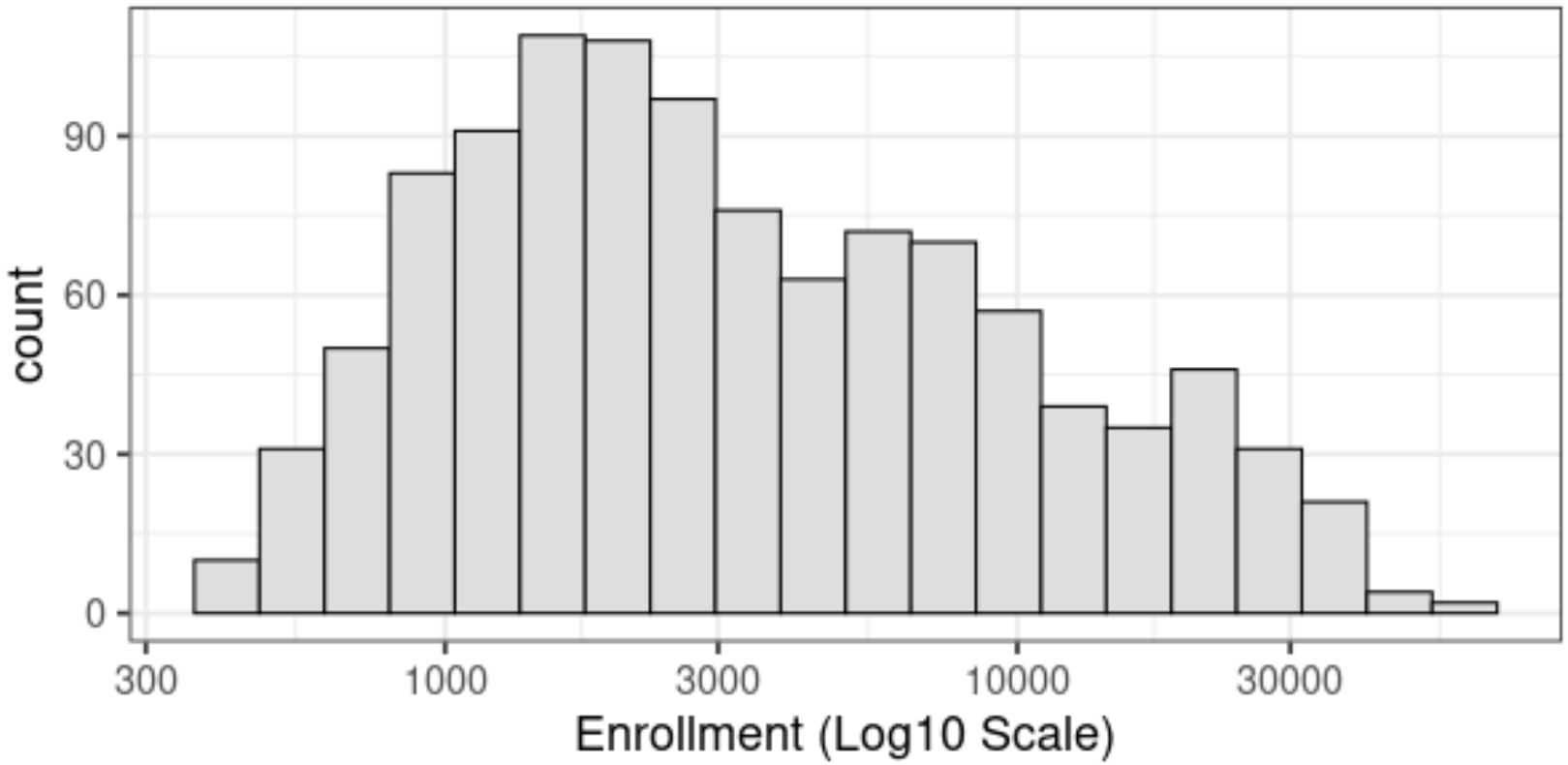
Comparing Mean with Median



Practice

For each of the following variables visualized below:

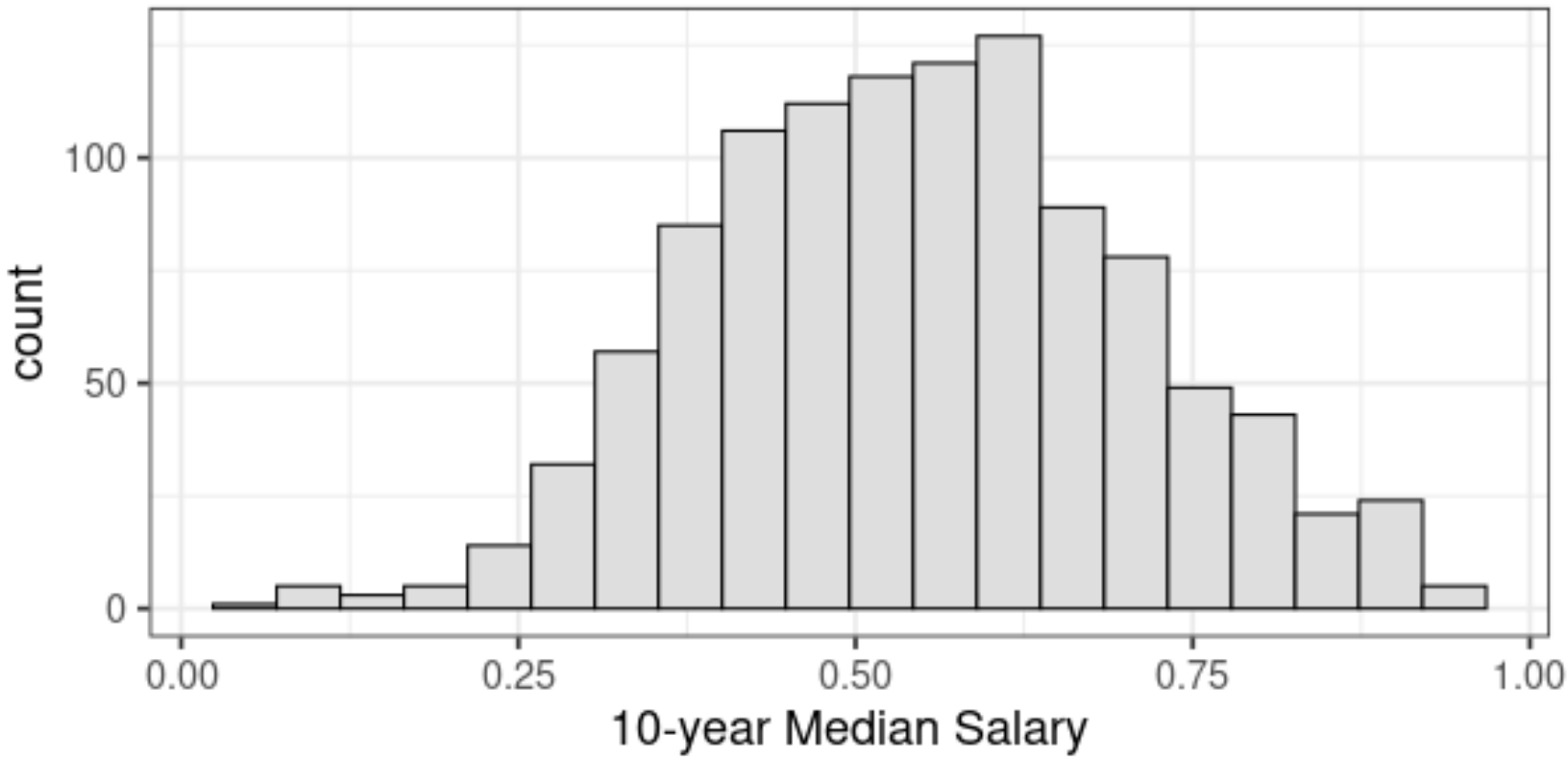
1. Determine approximate mean and median and which should be larger. How do you know?
2. Decide whether standard deviation or IQR is more appropriate for describing variability



Practice

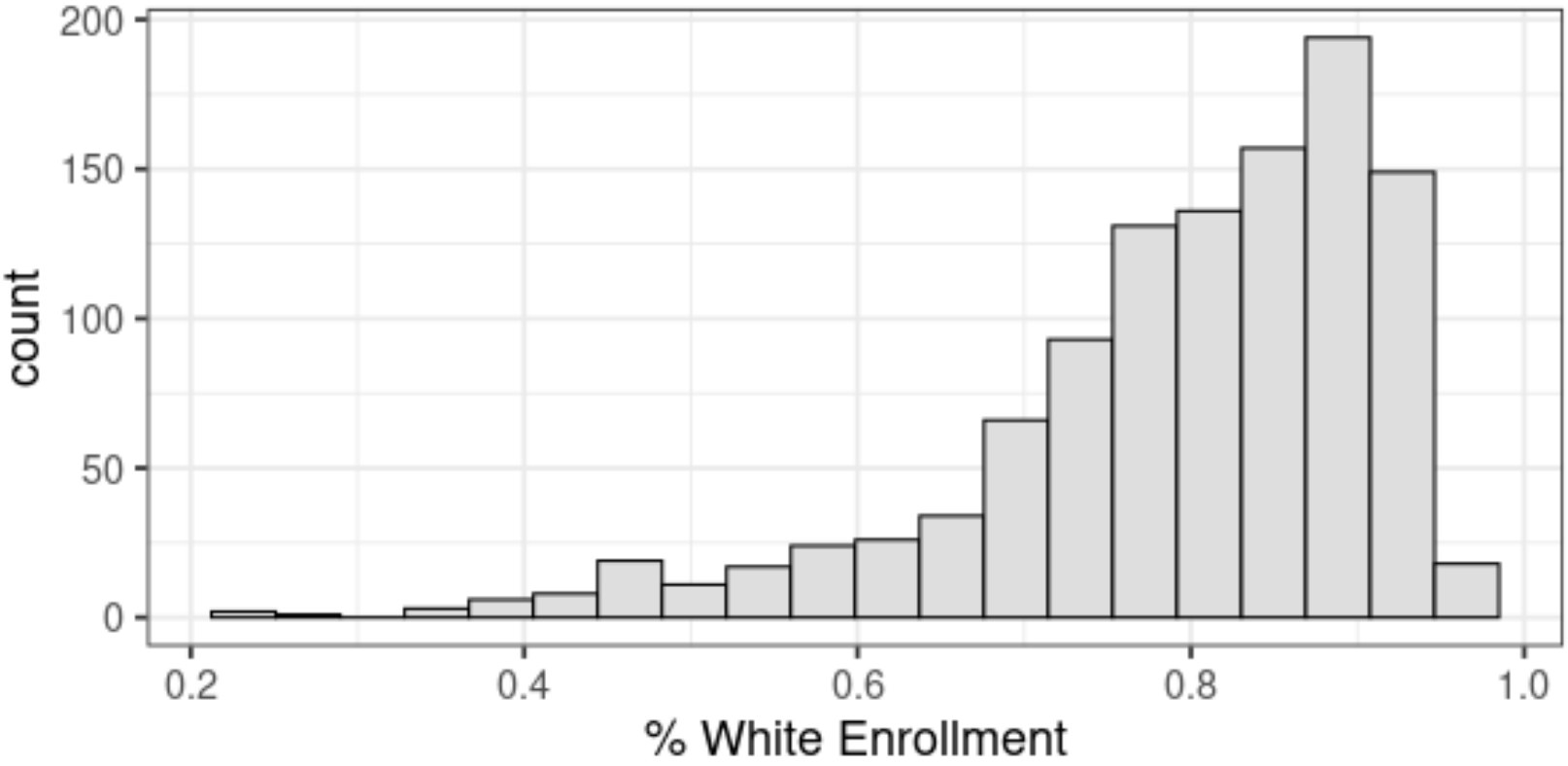
For each of the following variables visualized below:

1. Determine approximate mean and median. Are they very different?
2. Decide whether standard deviation or IQR is more appropriate for describing variability



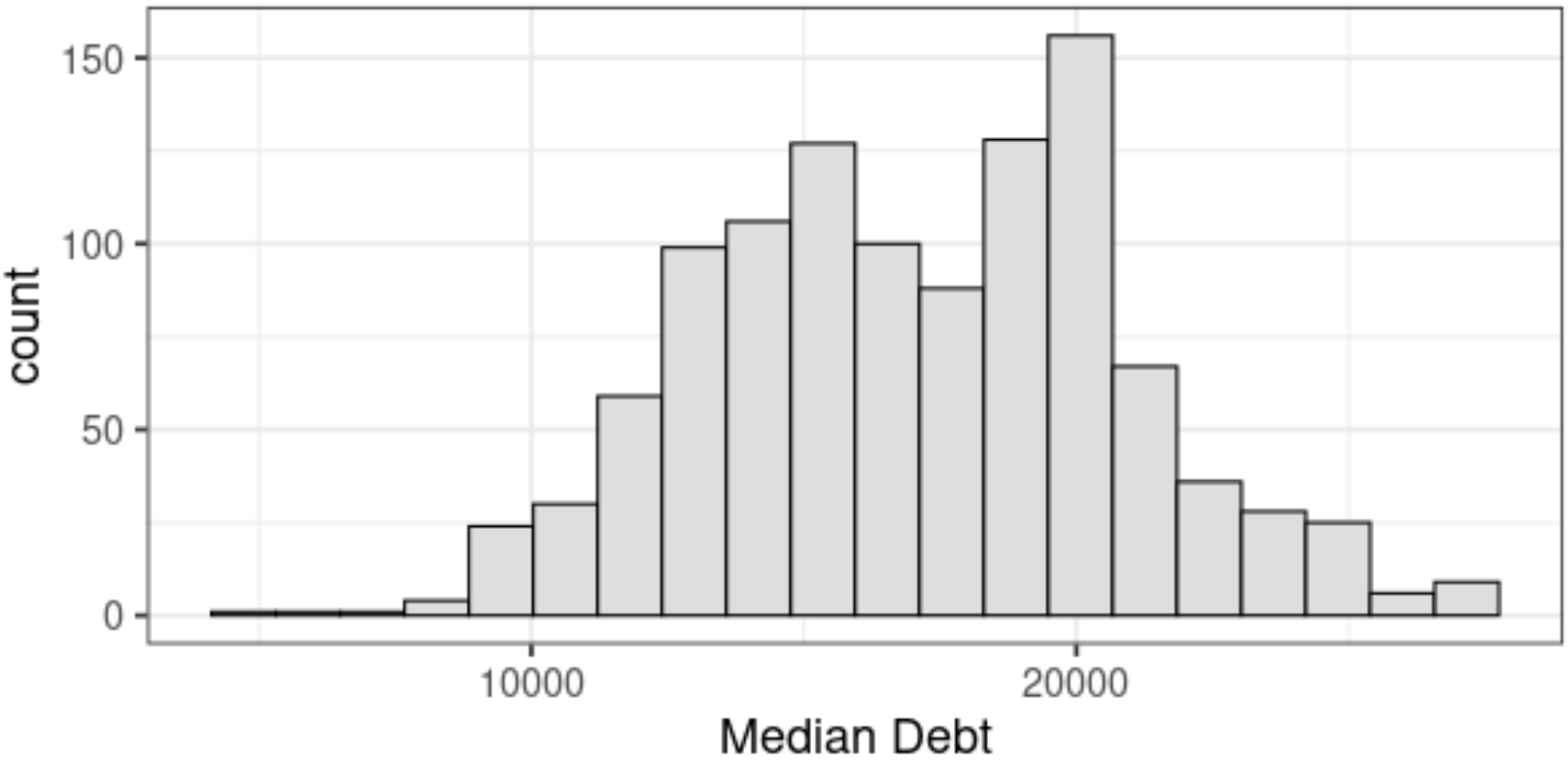
Practice

Describe the distribution of '% White Enrollment.'



Practice

Describe the distribution of Median Debt.



Advantages and Disadvantages

Order Statistics

Advantages:

- Robust to outliers (besides range)
- A "better" center for skewed data
- No assumptions

Disadvantages:

- Discards most data
- Math properties that are...of limited use
- Statistically less powerful

Moment Statistics

Advantages:

- Utilizes all of the data
- Very useful math properties
- Higher power

Disadvantages:

- Utilizes all of the data
- Makes assumptions
- Sensitive to outliers
- Sensitive to skew

Comparing Quantitative Variables

We can use center and spread to compare distributions. We typically refer to measures of centrality when discussing association (eg a t-test for the equality of means of two groups).

Consider the five-number summary for our Enrollment size

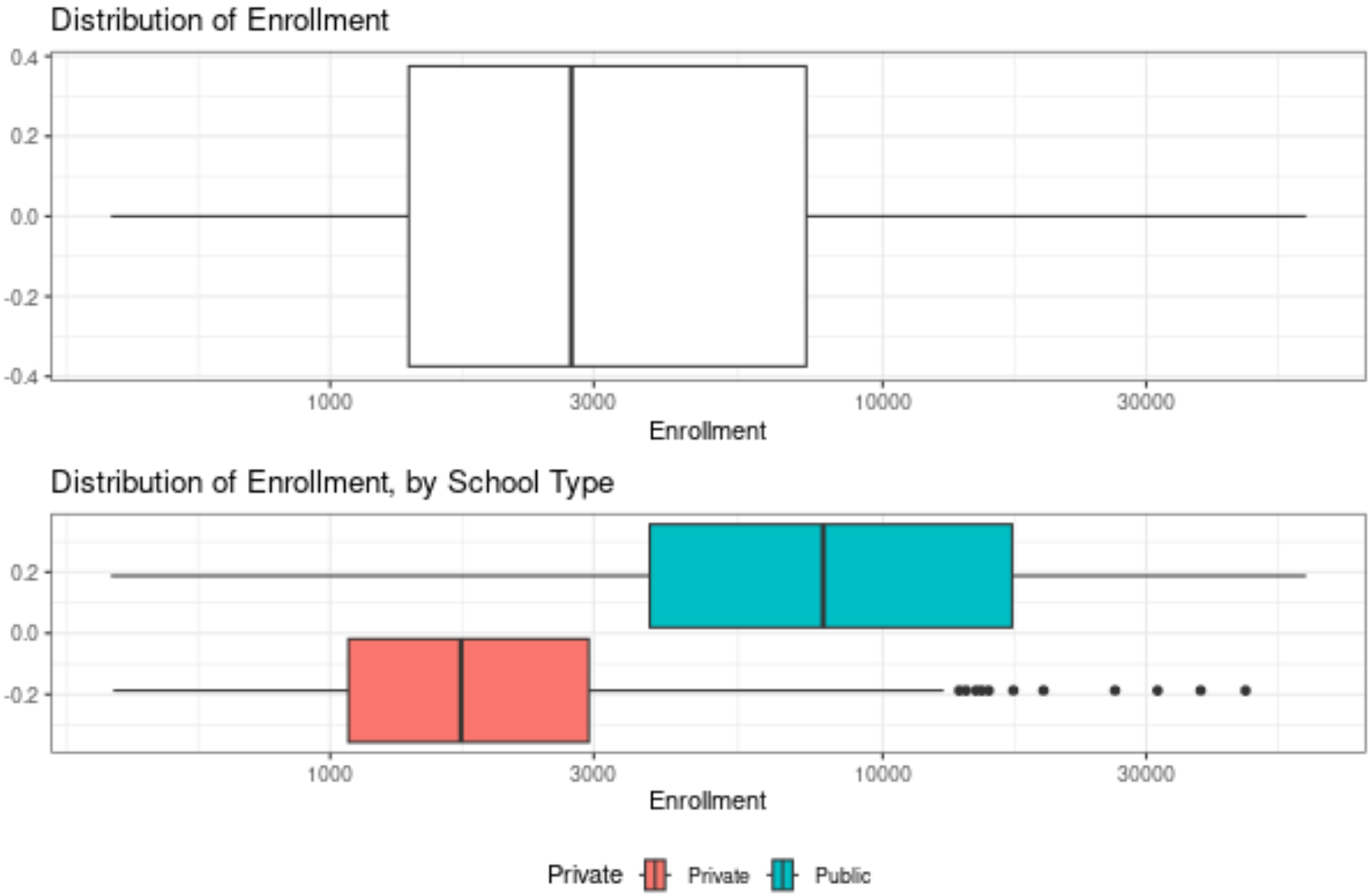
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
401	1388	2733	6241	7272	58392

Now lets look at the same variable, but add in info Public/Private

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Private	405	1079	1725	2720	2938	45370
Public	401	3788	7803	11325	17152	58392

Conditional Statistics (= Statistics for Each Category)

Which type of college tends to have more students enrolled? (center)



Parameter vs Statistics

It's important to distinguish between when we are talking about numeric summaries related to the sample or the population

Statistics are numerical summaries calculated from the *sample*.

- typically statistics are denoted using lowercase Latin alphabet characters
  - sample mean $= \overline{x}$
  - sample standard deviation $=$ s

Parameters are numerical summaries calculated from the population.

- typically parameters are denoted using Greek alphabet characters
  - population mean $= \mu$
  - population standard deviation $= \sigma$
- almost always the value of parameters are unknown to us

Parameter vs Statsitics

Statistics are to Samples what Parameters are to Populations Example:

Suppose we are interested to see the effects of daily asprin usage in younger aged people in increasing the years until the onset of the patients first cardiovascular event (eg heartattack). In particular, we want to increase the mean number of years. We randomly sample 500 people between the ages of 18 and 24 and gave them asprin daily. We then recorded the time from taking the asprin until their first cardiovascular event.

- Population
- Sample
- Parameter
- Statistic

Parameter vs Statistics

Statistics are to Samples what Parameters are to Populations Example:

Suppose we are interested to see the effects of daily asprin usage in younger aged people in increasing the years until the onset of the patients first cardiovascular event (eg heartattack). In particular, we want to increase the mean number of years. We randomly sample 500 people between the ages of 18 and 24 and gave them asprin daily. We then recorded the time from taking the asprin until their first cardiovascular event.

- Population: All "younger aged" people
- Sample: 500 people aged 18-24
- Parameter: mean years until the first cardiovascular event of all "younger aged" people
- Statistic: mean years until the first cardiovascular event of our sample